

The Indexical Character of Epistemic Modality

ABSTRACT: We assume a central thesis about modal auxiliaries due to Angelika Kratzer, the modal base presupposition: Natural language expressions that contain a modal component in their meaning, including all English modal auxiliaries and epistemic modal auxiliaries (EMA)s in particular, presuppose a modal base, a function that draws from context a relevant set of propositions which contribute to a premise-semantics for the modal. Accepting this thesis for EMAs leaves open (at least) the following two questions about the meaning of English EMAs like *must* and *might*: (i) What constraints, if any, are there on the character of the premise set for an EMA? And (ii) what is the nature of the relationship between premises and conclusion that is required for truth of the EMA statement? I argue for at least a partial answer to (i), with a hypothesis about constraints on the modal base for an EMA: EMAs, unlike some other types of modals, are indexical: They are anchored to an agent-in-a-situation whose doxastic state is currently under discussion in the context of utterance. Realized in a Kratzerian semantics, indexicality sheds new light on a number of outstanding puzzles, including the widely observed variability of anchoring of EMAs, the ways in which EMAs differ from so called root modals, Yalcin's (Mind 116:983–1026, 2007) puzzle (a version of Moore's paradox for epistemic modals embedded under attitudes), how to explain the apparent weakness of necessity EMAs, and problems with second order belief and disagreement.

1. Introduction¹

The influential work on modality in natural language due to Angelika Kratzer has contributed a thesis important for our understanding of Epistemic Modal Auxiliaries (EMAs) like *must* and *might* and other related epistemic expressions:²

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² See Kratzer (2012) for an overview of the development of this theory. Cf. also Veltmann's (1985) premise semantics.

THE MODAL BASE PRESUPPOSITION: Natural language expressions that contain a modal component in their meaning, including all English modal auxiliaries and EMAs in particular, presuppose a modal base, a function that draws from context a relevant set of propositions which contribute to a premise-semantics for the modal.

The modal base contextually determines a set of propositions; truth of EMA ϕ (*it must be raining*) will require that the EMA's prejacent ϕ (*it be raining*) be appropriately related to this premise set—e.g., follow from or be consistent with that set, or be probabilistically related to it in an appropriate way, etc. Thus, the modal base plays a crucial role in determining the domain of the modal operator associated with an EMA.

Accepting this thesis for EMAs leaves open (at least) the following two questions about the meaning of English EMAs like *must* and *might*:

- i. What constraints, if any, are there on the character of the premise set for an EMA, and how can this be reliably retrieved in the context of utterance?
- ii. What is the nature of the relationship between premises and conclusion that is required for truth of the EMA statement?

These questions are in principle independent. To accept the Modal Base presupposition, you don't have to agree with Kratzer's characterization of (human) necessity or possibility—with what logical relation between premises and “conclusion” (prejacent of the EMA) is required for truth.

Framed in the general Kratzerian approach in which the Modal Base presupposition is a central component, both these questions presently offer fruitful avenues of inquiry. I would argue that as usual for context sensitive expressions, in order to successfully address question (ii), you have to control for the factors that an answer to (i) should illuminate. In other words, you cannot get the semantics right until you properly control for context. Accordingly, here I will argue for a partial answer to (i).

I offer this hypothesis about the how the modal base for an EMA is constrained:

INDEXICALITY: EMAs, unlike other types of modals, are indexical: They are anchored to one of a limited set of agents whose doxastic (belief-based) perspectives are relevant in the discourse at the time of utterance.

Many have speculated that EMAs are indexical (Lyons 1977, Moltmann 2012, Portner 2009:181) or perspectival (Hacquard 2010, Kratzer 2020); one could take all Relativist accounts (e.g. Egan et al. 2005, MacFarlane 2005, Stephenson 2007) to argue that EMA interpretation depends on the perspective of a Judge. What is novel here is the way that this idea is realized formally as a set of inter-related presuppositions triggered by an EMA, focusing on English *must* and *might*. I adopt a new pragmatic tool, one independently motivated by a broader approach to indexicality: a type of discourse referent called a **discourse center**, modeled as an agent-at-a-time-in-a-situation, i.e. a centered world. The ways that discourse centers are made available and salient in context, tracked in a dynamic pragmatics, cannot generally be captured by binding in the logical forms of

individual sentences, e.g. as represented in the syntactic representation LF, and hence they are essentially indexical.

What does it mean for an EMA to be anchored to the doxastic perspective of an agent? Clearly some of that agent's beliefs will play a role in the interpretation of the EMA. But in addition, I accept a view argued for by von Stechow & Gillies (2010), Matthewson (2015), and Kratzer (2020): EMAs are evidential, in that what matters is not only what the agent knows generally, but a specific body of information available to that agent, information that is RELEVANT (in the sense of Roberts 1996/2012) in the context of utterance. Here is my understanding of Kratzer's (2020) characterization of evidentiality, the one I adopt here:

EVIDENTIALITY: The modal base for an EMA is evidential, yielding the factual evidence actually available to (though not necessarily recognized by) the anchoring agent, further constrained by what she does know. Such evidence does not always conclusively support the truth of the prejacent. In the terminology current in the literature, the resulting semantics is weak, not strong, so that *must* ϕ does not entail ϕ .

I modify this characterization of evidentiality to require that the evidence that plays a role in restricting the domain of an epistemic modal must be RELEVANT to the Question Under Discussion (QUD) (Roberts 1996/2012), a requirement independently motivated by Moss (2015), which turns out to play an important role in accounting for the apparent weakness of *must*.

In keeping with the general Kratzerian approach to natural language modal auxiliaries, their semantic content can be very simple. I'll assume universal (*must*) or existential (*might*) quantification over the restricted domain of possible worlds. Much of the interesting work is done by other aspects of their semantic character: how they conventionally appeal to the context of utterance to retrieve crucial features of their meaning in that context. Then because discourse is rich and complex, so that different contexts may vary along many distinct parameters, the resulting attested patterns of interpretation accordingly display complex patterns of variation.

In §2, I characterize and offer independent motivations for the notions of a doxastic perspective and discourse centers. In §3 I give the proposed indexical-evidential semantics and pragmatics for EMAs. Then in §4, in order to explicate and support the proposed analysis, I consider several well-known puzzles about the interpretation of EMAs and argue that the account in §3 is empirically superior to others on offer in shedding light on these puzzles. The puzzles include:

- the widely observed variability of anchoring of EMAs (§4.1),
- the ways in which EMAs differ from so-called root modals (§4.2),
- Yalcin's (2007) puzzle, a version of Moore's paradox for epistemic modals embedded under attitudes (§4.3),
- how to explain the apparent weakness of necessity EMAs (§4.4), and
- problems with second order belief and disagreement (§4.5).

Finally, in §5, I offer conclusions.

2. Background: Perspective and doxastic centers

2.1 Doxastic perspectives and indexicality

Since our beliefs may change over time, in modeling belief we talk about the beliefs of an agent at a time in a world, an ordered pair I call a *doxastic center*. Such a set is, of course, a centered world. We can also talk about the beliefs of an agent in a particular situation, where a situation is understood to be a sub-part of a particular possible world.³ In that case, assuming the situation has a given temporal extension in its possible world, we can derive the belief-time and belief-world from the situation. Then a doxastic center is the ordered pair of the agent and the situation. I adopt the following terminology:

A **situation** s is a mereological sub-part of a particular world w over an interval t , so that $s \leq w$, $t = \text{temporal-extension}(s)$.

A **doxastic center** is the ordered pair of a doxastic agent a and a situation s : $\langle a, s \rangle$.

DOX is a function from a doxastic center to a doxastic perspective.

A **doxastic perspective** is the set of worlds accessible from a doxastic center:

$\text{DOX}(\langle a, s \rangle)$.

DOX is a modal accessibility relation, taking a doxastic center $\langle a, s \rangle$ to yield the worlds consistent with the beliefs of that agent a at the situation-time t in the situation's world w .⁴ Each world in the doxastic perspective is one which for all a believes at t in w , may be actual.

Sometimes I get lazy and talk about a doxastic center as an agent, when it's clear what the centering situation is.

To capture *de se* phenomena, we need the worlds in a doxastic perspective to be centered worlds. But we have no need to talk about the *de se* per se in the analysis below—the propositions of interest are what Ninan (2010) calls “boring centered propositions”. So I will simply talk about a center's doxastic perspective at a world as a set of simple possible worlds.

We say that a doxastic center $\langle a, s \rangle$ serves as **anchor** for the contextually given point of view associated with a perspectival expression, one whose interpretation depends on the doxastic perspective $\text{DOX}(\langle a, s \rangle)$.

A central claim in the present account is that at any given time in a discourse, there is a limited set of doxastic perspectives that are relevant and salient at that time. For example, as argued by Ninan (2010) and Stalnaker (2014), the Context Set is a doxastic perspective—the worlds in which all the purported common beliefs of the interlocutors in the discourse are true. Accordingly, the Context Set at the time of utterance, centered on the join of the interlocutors, is always a relevant doxastic perspective. Similarly, many attitude predicates report on the beliefs and other belief-based attitudes (knowledge, hopes, fears, expectations, etc.) of the agent of the attitude at the time at which the attitude state is reported to obtain, making that agent's doxastic

³ See the mereology of situations in Kratzer (1989), where situations are world-bound.

⁴ I adopt Stalnaker's (2008) constraints on DOX. He modifies Lewis' (1979) characterization of this relation over centered worlds in ways that do not bear on the current analysis.

perspective at that time relevant and salient while interpreting the complement of the attitude. We will say that in a given discourse at a given time the center of a salient perspective is one of the available **discourse centers** at that time.

In Roberts (2015) I argue that indexicals are always anchored to one of the salient discourse centers in this sense: the speaker at speech time for *I*, the addressee for *you*, their join for *we*, and the agent of an embedding attitude at the attitude time for shifted indexicals in languages which have those (see below). In language after language, the same limited set of potential doxastic centers are the only potential referents of indexicals. In work in progress (Roberts 2015), I argue that the same set of doxastic anchors plays a role in constraining the interpretations of other indexical expressions, including demonstratives, shifting tenses (like that in Korean), conventional implicatures (Amaral et al. 2007, Harris & Potts 2009, Roberts 2022b), and indexical locatives.

Why do I say that these contextually available possible antecedents for indexicals are doxastic? This is because in every case they display *de se* features. Let me illustrate with a couple of cases.

With respect to indexical locatives, Barlew (2017) argues that the same discourse centers serve to anchor deictic motion verbs, like English *come* and its near-equivalent translation in the Bantu language Bulu, *zu*. These verbs presuppose that the location of the intended destination of movement from one location to another is where the relevant agent believes (or fears, or hopes, or imagines. . .) herself to be, and not necessarily (*pace* all the preceding literature) where she actually is. One of his arguments involves a variation on a famous example of the *de se* due to Morgan (1970):

- (1) [Context: Last week, Chicago Cubs baseball player Ernie Banks was hit on the head. He is now a lucid amnesiac. After the accident, Ernie was transported to Boston to work with an amnesia specialist. For all he knows, he has never been to Chicago. He has been reading about the baseball player Ernie Banks, but does not realize that he is reading about himself. He reads a false report that President Obama was in Chicago 3 weeks ago and while there met Ernie Banks. The doctor later tells her friend:
 - a. Ernie believes that President Obama traveled to Chicago.
 - b. #Ernie believes that President Obama came to Chicago.⁵

In (1a), with non-indexical *traveled*, the doctor truthfully and felicitously reports the (false) propositional content of Banks' belief. But in (1b), the anchor of *come* is understood to be the agent of the attitude, Banks. For it to be felicitous, Banks would have to believe that he-himself was located at the destination of the motion event, Chicago, at the time of Obama's purported trip. Banks *was* there then, but the utterance is unacceptable because he does not believe it: the presupposed *de se* implication fails.

The same *de se* phenomenon is observed in personal indexicals cross-linguistically: 1st and 2nd person indexicals are always *de se*. You can only see this, of course, when they are embedded under attitudes. But in a wide range of unrelated languages, 1st or 2nd person indexicals get

⁵ I use '#' to indicate that the example (or the marked part of it) is infelicitous in the context in which it is uttered, implicating that the expression is grammatical and might be felicitous in another context.

shifted interpretations when embedded under attitudes, where they may take as antecedent the 3rd person agent of the attitude. This is illustrated by the schematic example (2), comparing the Amharic 1st person pronoun to English *I*.

- (2) Situation to be reported: John says: “I am a hero”.
- a. *Amharic*: John_i says that -*ññ*_i am a hero [-*ññ* the Amharic 1st person pronoun]
 - b. *English*: John_i says that he_i is a hero/*John_i says that I_i am a hero

Crucially, in (2a) and comparable examples in other languages with shifted indexicals, the shifted 1st person receives a *de se* interpretation, wherein John is aware of self-ascribing being a hero.

A wide variety of such phenomena support the generalization that indexicals are always anchored to the doxastic perspective of some salient agent.

There is a very limited range of ways in discourse that a doxastic perspective may be made sufficiently salient that its center can anchor an indexical:

- In discourse, the perspectives of speaker and addressee(s) and their joint perspective, the Context Set given by the common ground, are always salient.
- In the complement of an attitude predicate the perspective of the agent of the attitude (usually expressed by the embedding subject) is salient.⁶
- In FID (Free Indirect Discourse), a common literary style where the author adopts the point of view of one of her characters,⁷ that character’s perspective may anchor an indexical, including deictic motion *come*:

- (3) But with Mr. Ramsay bearing down on her, she could do nothing. Every time he approached - he was walking up and down the terrace - ruin approached, chaos approached. She could not paint She rejected one brush; she chose another. When would those children come? When would they all be off? she fidgeted . . .
- [Virginia Woolf *To the Lighthouse*, quoted in Doron (1991:52). Woolf is describing the inner state of her character Lily Briscoe.]

Here, though Woolf uses 3rd person to refer to Lily Briscoe, *come* is anchored to Briscoe, also tacitly understood to be the source of the judgment that the approach of Mr. Ramsay was “ruin. . .chaos”.

Crucially, Barlew argues that non-interlocutors who aren’t the agent of an embedding attitude or the perspectival anchor of FID are not acceptable anchors for *come*, even when they are contextually highly salient. So in (4), “despite the fact that the interlocutors are talking about Peyton Manning, he is not an acceptable anchor”:

⁶ For locatives, see Fillmore 1975, Hockett 1990, and Oshima’s 2006a,b “deictic perspective shift”. For shifted indexicals, Deal 2020 summarizes a large body of literature on many languages.

⁷ Doron 1991, Sharvit 2008, Eckardt 2015.

- (4) [Context: Ann and Beth are in New York. Beth says:]
Sarah is in Chicago right now. However, she is a Denver Broncos fan and loves
quarterback Peyton Manning who has a game there tomorrow. #Therefore, she is coming
to Denver tomorrow.

Note that Manning the individual is salient at the time of utterance of the last sentence; for example, he could felicitously serve as the antecedent for *he* in a different final sentence: *He always puts on a great show*. Furthermore, Manning presumably self-ascribes being located in Denver. However, in contrast with Lily Briscoe in (3), Manning's perspective *per se* is not under discussion, hence is neither salient nor relevant, and therefore he cannot be the anchor of *come*.

We find the same inventory of possible doxastic anchors for English EMAs *must* and *might*, as we will explore in more detail in §4.1 below. For now, consider the following:

EMAs also shift in a main clause in FID, just where we expect the perspective to shift to that of the third person agent whose internal state is being described, in (5) for both *now* and *might*:

- (5) John pondered his situation. Where was Clarissa now? She might be in New Orleans with Sidney. But she might be in Chicago.

And embedded EMAs display *de se* features, as argued by Stephenson (2007:Chapter 4):

- (6) [Context: Banks-the-amnesiac reads an accurate report, based on a recent interview with Banks-the-shortstop, that the player suspects that the Cubs managers think the player is past his prime and are planning to demote him to the minor leagues. The management has repeatedly, publicly denied this, and amnesiac-Banks takes them to be telling the truth—why would they get rid of one of the all-time great players? So amnesiac-Banks thinks player-Banks is just paranoid. One of the nurses reports to the doctor her discussion about this with the patient:]
Our patient Ernie thinks that the Cubs must be planning to send him down to the Minors.

Assume that *must* is anchored to patient-Banks, the agent of *thinks*, as usual when embedded under an attitude. In this context, not only should the pronoun *him* be interpreted non-*de se*—so far as the patient knows, this particular belief is not about himself—but in order for (6) to be true it would have to be the case that the modal anchor for *must* is also non-*de se*: the beliefs that form the ground of the conclusion associated with *must* should be those of the player, which are not the patient's own beliefs in the situation reported (even though they are one and the same person). But (6) is false in the context given, even with *him* given the non-*de se* interpretation. This is so despite the fact that it is apparently true historically (per Wikipedia) that at the time late in his career that the actual Banks suffered a concussion while playing for the Cubs, the management *was* contemplating getting rid of him, so that the prejacent of *must* in (6) was actually true at the time it was uttered. I conclude that an EMA must be *de se* anchored to the doxastic state of some salient discourse center, in this case the agent of the attitude, so that the nurse is asserting (falsely) that it follows from the patient's beliefs that the Cubs will demote Banks-the-player. This is yet further evidence that EMAs are indexical.

In §3 we'll see how this doxastic anchoring bears on the semantics of EMAS.

2.2 Discourse centers and CHARACTER

The work on indexicality cited in §2.1 provides evidence that interlocutors regularly track the set of doxastic perspectives relevant in discourse at a given time, the set of **discourse centers** at that point, and that this is part of the information available in the context of utterance. We will model these discourse centers with a special type of **discourse referent**.

The notion of a discourse referent, going back to Karttunen (1976), is used in linguistics to track the available antecedents for anaphora in discourse. Heim (1982) modeled a discourse referent as a body of information that is purportedly about a single individual, who may or may not actually exist, or may be the arbitrary instantiation of the domain of a quantificational operator.

Technically, discourse referents are special variables, each bearing an index which serves as the address for tracking information about this purported entity. A discourse referent is **familiar** in a given context just in case all the interlocutors in the conversation are aware of its existence in that context; it may have been mentioned or correspond to an entity which has been the focus of attention of the interlocutors, or it's some entity whose existence is widely recognized (e.g., the sun).

A discourse referent has a “life-span”: if it is introduced, say, by an indefinite under the scope of an operator or as an arbitrary element of the domain of a quantificational operator, it is only a member of the set of discourse referents in the local context corresponding to the scope of the operator; i.e. its lifespan is restricted to that scope. So the set of available discourse referents is constantly changing across discourse; and this set is a part of the specification of the context of utterance at any given time, as in the following simplified characterization:

- (7) **Context of utterance** at a given point in discourse $D = \langle CS_D, QUD_D, DR_D, \mathbb{C}_D \rangle$:
- CS_D**: the Context Set, the set of worlds in $\cap CG$, CG the interlocutors' Common Ground: the set of propositions that they all purportedly take to be true.
 - QUD_D**: the set of questions under discussion (Roberts 1996/2012).
 - DR_D**: the set of familiar discourse referents (**dRefs**) at that point in discourse.
 - $T_D \subseteq DR_D$ is the set of temporal intervals discussed in D , at any given point in discourse containing the dynamically updated speech time t^* , so that for some $t_i \in T_D$, $t^* = t_i$.
 - $E_D \subseteq DR_D$ is the set of eventualities e discussed in D , each with its own time of occurrence, with $\text{time}(e) \in T_D$.
 - \mathbb{C}_D is the set of discourse centers $\mathbb{C}_{i,j}$, defined below.

The discourse referent ($dRef \in DR_D$) for an eventuality is interpreted model theoretically as a situation⁸—a temporally extended part of a particular possible world. An eventuality may be a state or a telic event; the aktionsart is indeterminate. When we assert that something occurs or

⁸ Cipria & Roberts (2000) argue that events are a type of situation. I use event dRefs here because Partee (1984) offers independent motivation for tracking them across discourse and relates them to Reference Time.

some state obtains, a dRef for that eventuality is introduced to E_D , as in Partee (1984). A modal or an attitude involves an operator which is a relation over worlds or situations, and so, like other quantificational operators, ranges over arbitrary elements of its domain. In a modal or attitude domain, the arbitrary eventuality e in its domain is introduced to E_D while interpreting the scope of the operator, and its lifespan is the scope of the operator under which it is triggered.

A discourse center © is a distinguished type of dRef, introduced in conjunction with consideration of an attitude, and labeled with the attitude-type. It consists of an ordered pair, its first element the agent of the attitude that triggers its introduction, its second the arbitrary eventuality e introduced by the attitude, from which we can retrieve $\text{time}(e) \in T_D$.

The entire discourse may be understood as (by default) a representation of the purported beliefs or belief-like states of the interlocutors, including their common ground CG, which is thus a type of Ninan/Stalnaker attitude. Those worlds in which the CG is true constitute the context set CS. Accordingly, throughout the discourse we have corresponding ©s for the interlocutors whose purported beliefs are being discussed. Some stretches of discourse are intended to be understood relative to a distinct attitude, that of some specific or arbitrary third party (non-interlocutor); these then constitute passages of FID. A question or instruction or a directive may be understood to query the addressee about her attitude toward the given question, or propose an attitude to the addressee, shifting the center to the addressee alone.

As usual in dynamic accounts of anaphora resolution (Kamp & Reyle 1993, Heim 1982, and subsequent work), the context for interpretation of a given constituent may be either **global**—in which case it's the context at the outset of utterance, or **local**. The global context serves as the local context for interpretation of, say, the first conjunct of a conjunction or the domain restriction of an operator. But then the global context is updated with the content of the first conjunct or of the operator's restrictive clause, to yield the local context for interpretation of the second conjunct/the operator's scope. Roughly, the resulting proposition expressed by the whole utterance is then added to the initial global context to yield a new global context for interpretation for the next utterance.

With the assumption of this kind of dynamic context update, we can define the set of discourse centers in a given context D as follows:

(8) The set of **discourse centers in D**, ©_D :

A discourse center © is an ordered set $\langle d_i, e_j \rangle$, such that $d_i, e_j \in DR_D$ and d_i is a doxastic agent whose beliefs in $e_j \in E_D$ are under discussion in D at the speech time in the actual world.

©_D is an ordered set. It always includes as its last element the unordered set of discourse centers involving the interlocutors $\{ \text{©}_{i,j}^*, \text{©}_{k,j}^@, \text{©}^{*+@} \}$, where:

- $\text{©}_{i,j}^*$ is a distinguished center corresponding to the speaker d_i in the actual event of utterance $e^* = e_j$,
- $\text{©}_{k,j}^@$ corresponds to the addressee d_k in $e^* = e_j$,

- $\textcircled{C}_{i+k,j}^{\text{CS}}$ ($= \textcircled{C}_{i+k,j}^{*+@}$) corresponds to inclusive ‘we’: the joint (purported) doxastic point of view of speaker and addressee $d_i \oplus d_k$ in $e^* = e_j$, as reflected in CS_D .⁹

Other elements of \textcircled{C}_D are discourse centers introduced in conjunction with the interlocutors’ direct consideration of alternative doxastic states, i.e. attitudes:

- Under the scope of an attitude predicate *pred*, $\textcircled{C}_{i,j}^{\text{pred}}$ (e.g., $\textcircled{C}^{\text{know}}$, $\textcircled{C}^{\text{believe}}$, $\textcircled{C}^{\text{hope}}$, $\textcircled{C}^{\text{claim}}$, etc.) is introduced as the first element of the local \textcircled{C}_D . $\textcircled{C}^{\text{pred}}$ will be the ordered pair of the agent d_i of the attitude in the arbitrary eventuality e_j in which it obtains.
- In FID, reflecting the doxastic perspective of some third party, a new first element is introduced: $\textcircled{C}_{i,j}^{\text{FID}}$, the agent d_i whose perspective in a given (possibly fictional) eventuality e_j is being adopted globally in the narrative.¹⁰
- Perspectival adverbials like *according to a* (at time t) or *from a’s point of view* (at time t) introduce a new first element $\textcircled{C}_{i,j}^{\text{POV}} = \langle a, e \rangle$, d_i representing the agent a in the (possibly arbitrary) eventuality e_j where the point of view is available at $t = \text{time}(e_j)$.
- In a conditional with an epistemic interpretation, we follow Kratzer in taking the *if*-clause to enrich a presupposed modal base. The modal base MB for such a conditional (as we will see in the next section) introduces a doxastic perspective, call it $\textcircled{C}_{i,j}^{\text{episMB}} = \langle d_i, e_j^{\text{episMB}} \rangle$, d_i the anchor of the epistemic modality, e_j^{episMB} the arbitrary eventuality where $\cap \text{MB}$ is true. Then the *if*-clause introduces a modified perspective, with the same anchoring agent but a new arbitrary eventuality, one where the content of the *if*-clause is true as well as that given by the modal base: $\textcircled{C}_{i,j}^{\text{if}} = \langle d_i, e_j^{\text{episMB+if}} \rangle$.

When the scope of the operator triggering introduction of some \textcircled{C} is closed off, \textcircled{C} itself is removed from \textcircled{C}_D .

I know of no other way in which discourse centers are introduced to \textcircled{C}_D in English.

The order over discourse referents in \textcircled{C}_D reflects the relative scope of any operators which introduce elements other than the set of interlocutors at its bottom, playing a role in relative salience of the available discourse centers. The order is used to track which perspective is most recently introduced at any given point in discourse: When a new $\textcircled{C}_{i,j}$ is introduced, it is the most local and is first in the order; as with discourse referents generally, it is removed at the end of its lifespan (at the limit of the scope of the operator under which it was introduced), even if the agent d_i itself is globally accessible (remains in DR_D). With multiple embeddings of attitudes, this predicts the following type of update sequence:

- (9) [The doctor is reporting what the nurse told him in (6):]
 According to the nurse₄ [_{S1} Ernie₇ thinks [_{S2} the Cubs must plan to demote him]]
 prior (global) context: $\{ \dots d_4, d_7 \dots \} \subseteq \text{DR}_D$, $d_4 = \text{nurse}$, $d_7 = \text{EB}$, $\textcircled{C}^* = \langle \text{doctor}, e^* \rangle$,
 $\textcircled{C}^@ = \langle \text{addressee}, e^* \rangle$, $\textcircled{C}_D = \langle \{ \textcircled{C}^*, \textcircled{C}^@ \} \rangle$

⁹ where \oplus is the join operator of Link (1983).

¹⁰ There is a phenomenon much like FID, occurring in the complements of certain attitude verbs and licensing a shift in perspective, with some indexicals shifting accordingly. Abrusán (2021) calls this *protagonist projection*, and argues that it differs from FID in important respects. Protagonist projection is captured in (8) by the introduction of $\textcircled{C}^{\text{pred}}$ for the relevant verbs. I take it that FID is always and only introduced globally.

update under *according*: $\mathbb{C}_D = \langle \mathbb{C}^{according}, \{\mathbb{C}^*, \mathbb{C}@\} \rangle$, where $\mathbb{C}^{according} = \langle d_4, e^{according} \rangle$
 update under *thinks*: $\mathbb{C}_D = \langle \mathbb{C}^{think}, \mathbb{C}^{according}, \{\mathbb{C}^*, \mathbb{C}@\} \rangle$, $\mathbb{C}^{thinks} = \langle d_7, e^{think} \rangle$
 final update: $\mathbb{C}_D = \langle \{\mathbb{C}^*, \mathbb{C}@\} \rangle$, $\mathbb{C}^* = \langle \text{doctor}, e^* \rangle$ if doctor keeps talking

Think of the eventualities that are introduced by attitudes as perspectival situations, those reflecting the relevant perspective of the anchoring agent. $\mathbb{C}^{according}$ is the perspective associated with the purported belief state of the complement of *according to*.¹¹

More generally, the set of available discourse centers \mathbb{C}_D is updated regularly over time as (a) speakers and addressees change role, (b) speech time t^* and utterance situation e^* change, and (c) as we just saw for (9), we enter or leave the scope of an embedding attitude, introducing a corresponding locally relevant $\mathbb{C}^{attitude}$. The order reflects the way that scopes are nested: one can only remove a \mathbb{C} from the top of the order. This dynamic bookkeeping mechanism has a predecessor in the way that Reichenbachian Reference Time is modeled in Partee (1984), where it changes as the interlocutors move from talking about one eventuality to another.

A final preliminary: The conventional content of an expression is captured in its **CHARACTER**. This notion differs from Kaplan's (1977) Character because in dynamic pragmatics context may be updated in the course of interpretation, as just described. So indexical presuppositions needn't always be satisfied by the global context of utterance, as in Kaplan's account, but may (with lexical restrictions) be merely locally satisfied (Roberts 2015). Thus, an embedded indexical may have its presupposition of an indexical anchor satisfied not in the global context of utterance, but merely locally under attitudes, as when *come* or *must* or shifting personal indexicals are embedded.

- (10) The **CHARACTER** of an expression involves (at least) the following types of content:
Presupposed content: that which constrains the local contexts in which utterance of the expression is felicitous
Semantic content: that which enters into the compositionally calculated truth conditional content of any constituent in which the expression occurs—its conventional contribution to what is asserted, asked or directed

3. Semantics and pragmatics for epistemic modals

An EMA like *must* or *might* is:

- (a) **indexical**, presuppositionally anchored to a discourse center;
- (b) **evidential**, presupposing a particular type of evidential modal base, the body of evidence actually available to the anchor in the anchoring situation, enriched by the anchor's doxastic state; and

¹¹ Cf. see Potts' (2007) analysis of the meaning of *say* in indirect discourse, where the content of the complement is something the speaker is committed to, a purported belief state. Hence, arguably *say* does denote an attitude, rather like the purported beliefs of \mathbb{C}^{CS} . And it seems that *according to* has a content like that of *say*. Like *say*, *according to* may report characterizations of a counterfactual situation in which the reporting agent purportedly finds herself.

- (c) **inferential**, in that its semantic content is that the prejacent p follows from or is consistent with the relevant body of information.

I offer evidence for (a) and (c) in section 4 below. (a) will be captured in the way that use of an EMA presupposes an anchoring discourse center. (c) will be reflected in the proposed semantic value of *must* and *might*, below. (b) will be captured here in the type of modal base presupposed by an EMA.

In taking EMAs to be evidential, I agree with many authors in the literature, who offer their own evidence for this claim. But it isn't clear that everyone means the same thing by *evidential*. For example, von Stechow & Gillies (2010) take *must* to be evidential and strong, so that, in effect, the evidence which forms part of the basis for concluding that the prejacent is true must itself be true in the world of evaluation. But Matthewson (2015) disputes this, arguing that some of the evidence supporting an EMA claim may be false or misleading. Kratzer (2020) offers several examples where some of the relevant evidence is false. And she concludes that rather than being properly epistemic, the modal base of an EMA is based on weak belief (Hawthorne, Rothschild & Spectre 2015; Rothschild 2020).

But Kratzer (2020) would in effect enforce Lewis' (1996: 554-55) "rule of Actuality": "The possibility that actually obtains is never properly ignored." An agent cannot properly ignore a possibility "that evidence and arguments justify him in believing—whether or not he does so believe." Kratzer formalizes Lewis's intuition by requiring that EMAs display factual domain projection (Kratzer 2002):

(11) Factual Domain Projection

For any part of a world s , $\text{fact}(s)$ is the set of possible worlds that have an exact match of s .

The evident facts about the situation of an EMA anchor should all be brought to bear on what can be concluded. We see a reflex of this in the following example, pointed out to me by Stewart Shapiro (p.c.), describing a scene in the Fox TV show *Monk*:¹²

(12) [Context: A young and inexperienced detective is examining the body of a woman who fell from a balcony 21 stories above, while senior investigator Monk looks on:]

Detective: It might be murder, or it might be suicide.

Monk: No, it cannot be suicide. Look at her toenails. She was painting them and did not finish that one. So she was interrupted.

Let's say that Monk and the detective have the same body of perceptually available information about the corpse. Visual inspection makes available the information that it appears to be the body of a young woman wearing jeans and a t-shirt, slender and brunette, and that she has one toenail half-painted and another not done at all. In this evidential situation, available to both, Monk notices this last bit of information and takes it to be relevant. The younger detective overlooks this crucial detail or fails to see its relevance, leading to a premature conclusion. Note that there

¹² from episode 8 of Season 1: "Mr. Monk and the Marathon Man"

is a lot about the situation that is not epistemically available to the detective or Monk: For example, they can't tell whether the young woman's blood shows evidence of poison, etc.

Intuitively, we want the domain of the detective's *might* to range over worlds in which all the available evidence is true. In that sense, the domain is factual and not properly epistemic: As in the story, since some of the available information has not been noticed by the agent, arguably it is not known by him. Thus, the grounds for an EMA claim may go beyond the epistemic/doxastic state of the anchoring agent.

Summarizing what we have seen so far:

- (13) **Evidential modal base** [Preliminary version]
 E^{vid} is a function from centered situations to sets of propositions s.t. for any agent a , situation s , $E^{\text{vid}}(\langle a, s \rangle)$ yields the set of propositions which exactly characterize a 's evidential circumstances in s : the information actually available to a in s , in whatever way it is made available.

So defined, $\cap E^{\text{vid}}(\langle a, s \rangle) \subseteq \text{Fact}(s)$, the set of worlds that include exact replicas of a 's evidential situation s . But as Kratzer is careful to emphasize, one's information in a given situation may not all be of the same quality: not only may it come to one more or less directly, it may also be more or less reliable, some of it even false—as with unreliable hearsay or hallucination. Thus, it is not necessarily the case that the world of which s is part is itself in $\cap E^{\text{vid}}(\langle a, s \rangle)$, as argued independently by Matthewson (2015).¹³

In fact, there are occasions when the complete body of evidence one might take into account in making a judgment doesn't appear to be internally consistent. Consider the following variant on an example due to von Fintel & Gillies (2010):

- (14) [Context: Immediately after a shooting in a dead-end alley, the police found the victim dead, the murder weapon thrown to the side of the alley, and two men, John and James, each claiming to be an innocent bystander. A woman was watching out her window several stories above. There were no prints but some DNA from saliva on the gun. A few days later, one detective says to the District Attorney:]
Given the results of the DNA tests, John might be the murderer. But if we take the eyewitness seriously, it must be James.

Even supposing that the eyewitness is honest, it is well-known that one's interpretation of events witnessed may be skewed by misperception (the "gun" James held was really a cell phone), prejudice (James was African American), oversight (James was close to the victim trying to help,

¹³ Kratzer (2020) explains that the evidential domain of an EMA is **factual** not in that all of the available evidence is true in the world in which the agent actually finds themselves, but in that whatever the evidence, its source, and its reliability, this is replicated in all the worlds in the modal's domain. And she is at pains to make clear that the evidential situation needn't contain the actual things the agent has information *about* in that situation—in (12) it needn't include the actual corpse—so that the available information pertaining to such an object may be limited—the detectives don't have access to information about whether she was poisoned. But crucially, it includes all the RELEVANT available evidence, noticed or not.

while the fatal shot was fired from a distance in the shadows), or poor memory. But there is also an emerging realization that DNA evidence itself depends on fallible human interpretation of the test results, and is thus inconclusive (Shaer 2016). So the detectives can be pretty certain that one of these conflicting pieces of evidence is misleading, but they can't say which. Nonetheless, they seem correct in drawing the EMA conclusions that they do, and it seems that they could *not* draw those conclusions only from the facts they are sure about.

We need to restrict the domain of an EMA to yield a consistent body of evidence, in order to avoid *ex falso quodlibet*. Here's a simple (perhaps too simple) way one might do this: Take the modal's domain to be the set of worlds in which is true the largest consistent set of propositions reflecting the evidence. In order to consider the implications of the apparently inconsistent information, as in (14), explicitly add as much of that information to the modal base (in (14) with the *given* or *if* clauses) as possible while still maintaining consistency.

There is yet another refinement of (13) to consider. The notion of evidence is relational, as reflected in typical dictionary definitions: Evidence is "something that furnishes proof" (*Merriam-Webster* on-line) or "the available body of facts or information indicating whether a belief or proposition is true or valid" (*Oxford Languages* on-line on Google). As in (12), in (14), the evidence includes anything in that situation which bears on the cause of death. Part of the difficulty in any crime situation lies in determining what about the situation is relevant and what is not. Roberts' (1996/2012) notion of RELEVANCE to the QUD offers us one way of approaching this issue: Only those bits of information that would contextually entail a partial answer to the QUD should be considered: in (12) those which plausibly bear on *how did the woman die?*, in (14) on *who is the killer?*.¹⁴

- (15) An utterance u is **RELEVANT** in a context D just in case it addresses the $QUD_D q$, i.e. contextually entails a partial answer to q (u is an assertion), is part of a strategy to answer q (m is a question), or suggests an action to the addressee which, if carried out, might help to resolve q (m is a direction).

So we have the following, with additions to (13) to guarantee consistency and relevance:

- (16) **Evidential modal base** [Revised final version]
 E^{vid} is a function from centered situations to sets of propositions s.t. for any agent a , situation s , $E^{vid}(\langle a, s \rangle)$ yields the maximal consistent intersection of the set of subsets of the set of propositions which exactly characterize a 's RELEVANT evidential circumstances in s : the information available to a in s , in whatever way it is made available.

An agent's evidential state $E^{vid}(\langle a, s \rangle)$ may differ from their belief state in the same situation. For example, in (12), the worlds consistent with the detective's belief state include some where the toenails were all painted, while in all the worlds accessible via the evidential modal base, the corpse had the actual unfinished pedicure. But when we reason on the basis of our evidence, we

¹⁴ This echoes the findings of Moss (2015), who argues that the QUD in a given context of utterance, a partition over worlds in the interlocutors' Common Ground, plays an important role in domain restriction of the epistemic vocabulary she considers, and so ultimately in their truth conditional contributions, including explaining apparent non-commutativity of disjunctions containing epistemic vocabulary.

typically bring to bear not only the relevant evidence, but also more general knowledge and background information. Accordingly, we will enrich the evidential modal base for an EMA with the anchor's actual belief state in the world of interpretation, so that the modal base for an EMA will be doxastic as well as evidential.¹⁵

What if the available evidence appears to be inconsistent with what one believes? Typically, if the evidence appears to support p and one has theretofore believed $not-p$, either one revises one's beliefs to take p to be true or else one treats the problematic evidence as weak and hence explicitly hypothetical or even counterfactual, entertaining it with an *if*-clause as in the contradictory evidence case (14) above.

Finally, as Lewis (1996) discusses, when we reason about knowledge we don't typically take into consideration all the most outlandish possibilities, those that only the most skeptical epistemologists consider, like worlds in which we're brains in a vat. So, without actual evidence to the contrary, we typically exclude such abnormal worlds from the domains of epistemic modals. For this, Kratzer (1981) introduces another conversational background, her ordering source g . For EMAs we'll use a special kind of ordering source, N^{orm} :

- (17) **Normal conversational background, N^{orm}** : a function from centered situations to sets of propositions s.t. for any $\langle a, s \rangle$:
 $N^{orm}(\langle a, s \rangle)$ yields the set of propositions that characterize what a in s takes to be the way things normally are in the world w such that $s \leq w$.

Hence, $\cap N^{orm}(\langle a, s \rangle)$ is the set of normal worlds from the perspective of a in s . Notice that if the ordering source is based on accurate knowledge about what's normal, the propositions it contains are not themselves guaranteed to be true in a given situation in the world in which the agent actually finds herself: Actual situations aren't always normal.

As usual in Kratzerian modal semantics, we use the set of propositions given by an ordering source to induce an order over the set of worlds given by the modal base:

- (18) For any set of propositions G , worlds w, w'
 $w \leq_G w'$ iff $\{p: p \in G \ \& \ w' \in p\} \subseteq \{p: p \in G \ \& \ w \in p\}$

If the set of G -propositions true in w' is a proper subset of those true in w , w is more G -like than w' .

Then, again as usual in a Kratzerian premise semantics, the domain of a modal auxiliary is determined by the modal base and ordering source, yielding the most normal worlds in which the propositions given by the modal base are true, as follows:

- (19) For any f, g, \odot , $\mathbf{Dom}_{f,g}(\odot) = \{w' : w' \in \cap f(\odot) \ \& \ \forall w'' \in \cap f(\odot): w' \leq_{g(\odot)} w''\}$

¹⁵ One might further restrict the domain to only consider those propositions that the anchor *knows*, rather than just those that she believes to be true, as proposed by McCready (2008,2010). I think this is too strong, as others have argued in the literature.

Assuming the definitions in (16) – (19), we can now propose a semantico-pragmatic CHARACTER for *must*:

(20) **CHARACTER of English epistemic *must*_{i,j}^{f,g}:**

Given context of utterance $D = \langle CS_D, DR_D, \odot_D \rangle$, with situation of evaluation e_j :

Presupposed content:

Indexical anchor: $\odot_{i,j} = \langle d_i, e_j \rangle \in \odot_D$.

Modal base f s.t. $f(\odot_{i,j}) = E^{\text{vid}}(\odot_{i,j}) \cup \{p \mid \text{DOX}(\odot) \subseteq p \ \& \ E^{\text{vid}}(\odot_{i,j}) \cap p \neq \emptyset\}$

Ordering source g s.t. $g(\odot_{i,j}) = N^{\text{orm}}(\odot_{i,j})$

Semantic content: $\lambda p_{\langle s, t \rangle} \lambda w . \text{Dom}_{f,g}(\odot_{i,j}) \subseteq p$

The CHARACTER of *must* is indexical because it is anchored to a presupposed discourse center $\odot_{i,j}$, crucial for retrieving the presupposed modal base f and ordering source g . In general, anaphora resolution picks out the most salient available antecedent that yields a consistent, coherent and RELEVANT interpretation, and I assume that the same obtains for discourse center antecedents, so that no special stipulation to that effect need be introduced into the indexical anchor presupposition. The conditions on f tell us that it depends upon a body of RELEVANT evidence actually available to the anchor plus the anchor's general belief state, as consistent. Then the domain given by f and g is the set of most normal worlds consistent with the agent's belief state in which this evidence is true.

The semantic content of *must* is simple: The prejacent is true in all the worlds consistent with the center's normalized evidential doxastic perspective, the presuppositionally restricted domain. As noted in the introduction, I do not offer evidence for this simple semantics; but it will suffice to show the utility of the presupposed content that is proposed here. This semantic content makes *must* an inferential evidential: As evidence for the truth of the prejacent it proffers that the prejacent can be inferred from the relevant evidence available to the anchoring center.

But in the end, both because the anchor cannot be sure either that none of the evidence is misleading or that the actual world is normal, the domain given by (19) may not include the actual world of evaluation. Thus, ***must* ϕ does not entail ϕ** . The modal is factual but weak, as we'll discuss further in §4.4.

Here is a comparable semantics for epistemic *might*, differing, as usual, only in that its semantic content merely requires that the prejacent be consistent with the domain, instead of being entailed by it:

(21) **CHARACTER of English epistemic *might*_{i,j}^{f,g}:**

Given context of utterance $D = \langle CS_D, DR_D, \odot_D \rangle$, with situation of evaluation e_j :

Presupposed content:

Indexical anchor: $\odot_{i,j} = \langle d_i, e_j \rangle \in \odot_D$.

Modal base f s.t. $f(\odot_{i,j}) = E^{\text{vid}}(\odot_{i,j}) \cup \{p \mid \text{DOX}(\odot) \subseteq p \ \& \ E^{\text{vid}}(\odot_{i,j}) \cap p \neq \emptyset\}$

Ordering source g s.t. $g(\odot_{i,j}) = N^{\text{orm}}(\odot_{i,j})$

Semantic content: $\lambda p_{\langle s, t \rangle} \lambda w . \text{Dom}_{f,g}(\odot_{i,j}) \cap p \neq \emptyset$

4. Application to some puzzles about the meanings of EMAs

4.1. Variable anchoring

There has been a good deal of debate about whose epistemic state can be appealed to in the semantics of epistemic modals.¹⁶ Everyone notes that there is at least a default tendency to understand the speaker to be the relevant agent whose epistemic state is at-issue. DeRose (1991:5) proposes a *speaker inclusion constraint*:

Speaker inclusion constraint: the relevant community [must] include the speaker. Hence “whenever S truly utters *a might be F*, S does not know that *a* is not *F*.”

As discussed by von Fintel & Gillies (2007, 2011), and illustrated in (5) from §2, this requirement is too restrictive:

- (5) John pondered his situation. Where was Clarissa now? She might be in New Orleans with Sidney. But she might be in Chicago.

In (20) and (21), an EMA presupposes both an anchor \odot , and a modal base f whose content is constrained by \odot . Both presuppositions are anaphoric, requiring that we locate an available f and \odot , \odot to lend its doxastic perspective to the domain via f . Domain restriction is essentially anaphoric (von Fintel 1994), so here the anaphoricity of f and \odot is unexceptional. And anaphora resolution is an essentially pragmatic phenomenon: With the exception of anaphors like *themselves* which must be syntactically bound in LF, in discourse an anaphoric antecedent is the most salient familiar discourse referent that satisfies the trigger’s presuppositions without yielding anomaly. The order on \odot_D in §2 is intended to capture both (scope-constrained) availability and relative salience of the available discourse centers: a given EMA will be understood to be anchored to the most salient member \odot of \odot_D , so long as that yields a felicitous interpretation. Thus, we make clear predictions about the available anchors for a given EMA in its context of utterance.

The speaker’s point of view at speech time t^* (that of the utterance situation e^*) is always the default, for all perspectival expressions. Especially in the out-of-the-blue context, the only reasonable way to anchor an EMA like (22) is with a center which includes the speaker:

- (22) John might be the thief.

However, nothing in the utterance itself determines whether this anchor should be \odot^* or some \odot which includes the speaker among others, e.g. \odot^{CS} whose joint (purported) doxastic state is CS_D . If the interlocutors are detectives considering a jointly available body of evidence, \odot^{CS} is the most likely anchor. If they’re arguing over what evidence to take seriously or the speaker is privy to information that the addressee lacks, \odot^* is more likely. Only context determines.

¹⁶ See Hacking (1967), Egan et al. (2005), MacFarlane (2005), Stephenson (2007), von Fintel & Gillies (2007, 2011), *inter alia*.

The anchor may be a group containing the speaker at some actual time other than t^* , as in (23), where the adverbial explicitly shifts the doxastic point of view to that of the speaker and others in the anaphorically retrieved denotation of *we* at some time t prior to t^* , $\langle d^{*\oplus}, e \rangle$, $e < e^*$:

- (23) Given what we knew at the time, John might have been the thief. [von Fintel & Gillies 2007, (4b)]

But there are cases where the grounds appealed to appear to be solely those of the addressee, $\textcircled{C}^@$:

- (24) Where might you have put the keys? [von Fintel & Gillies 2007, (14)]

As with evidential particles in interrogatives in many languages, this amounts to interrogative flip (Speas & Tenny 2003)—wherein the anchoring perspective naturally becomes that of the addressee when seeking information they might offer.

We see something similar in (25), where *your* and the imperative mood of the main clause, suggest that the anchor is the arbitrary group containing the addressee at some hypothetical future time:

- (25) [Military trainer:] Before you walk into an area where there are lots of high trees, if there might be snipers hiding in the branches use your flamethrowers to clear away the foliage. [after von Fintel & Iatridou 2003]

The *if*-clause constitutes a precondition for carrying out the instructions—they are applicable in circumstances in which the addressee’s available evidence in a combat scenario leaves open the possibility that there are snipers (Roberts 2018). Then the only reasonable anchor for *might* would be the trainee soldier(s) in such an actual (future) situation, who have to decide whether the correct conditions obtain for using their flamethrowers. Note that the imperative main clause can readily be switched out with a near-synonymous declarative: *if there might be snipers hiding in the branches you should use your flamethrowers to clear away the foliage*, yielding an assertion. But still *might* in the antecedent of the conditional is understood to be anchored to the arbitrary soldier. So the second person orientation in (25) is not due to the imperative *per se*. The key is that the whole passage is part of a complex set of instructions: how to behave in a combat situation. It is understood throughout that the realization of these instructions will call upon the judgment of the addressee *in that situation* at future time t ; and hence any epistemic modal in that passage is by default so-anchored.

Since it is essentially pragmatics which tells us which of $\textcircled{C}^{*(+)}$, $\textcircled{C}^{@(+)}$ or $\textcircled{C}^{\text{CS}}$ is the intended anchor for such EMAS in matrix clauses, the bottom element in \textcircled{C}_D is an unordered set. Which of the interlocutors yields the most plausible and RELEVANT interpretation depends on the preceding discourse. Below, for simplicity, I will ignore $\textcircled{C}^@$ and $\textcircled{C}^{\text{CS}}$ unless directly relevant.

Now consider examples like (5) (§2) involving FID. The hallmark of FID is that the central perspective is shifted from \textcircled{C}^* to that of some third party at a given time under discussion, $\textcircled{C}^{\text{FID}}$. Then those indexicals that are shiftable take $\textcircled{C}^{\text{FID}}$ as their anchors, including English *now*, *here*, a variety of discourse particles and exclamatives, predicates of personal taste, *come*, and as we saw

in (5), *must*; but not *I* or *you*.¹⁷ In other words, only some indexicals can shift in FID, and this shiftability is lexically given (as is common in languages with shiftable personal indexicals). In FID passages, $\textcircled{C}^{\text{FID}}$ is at the top of \textcircled{C}_D , the most salient global perspective, and all shiftable indexicals must shift, though there may be an embedding attitude *within* the FID: We could extend (5) as follows:

(5') John pondered his situation. Where was Clarissa now? She might be in New Orleans with Sidney. Steven insisted she must be in Chicago, but John was not convinced.

Under the scope of *insisted*, the local $\textcircled{C}_D = \langle \textcircled{C}^{\text{insist}}, \textcircled{C}^{\text{FID}}, \textcircled{C}^* \rangle$, where $\textcircled{C}^{\text{insist}} = \langle \text{Steven}, e^{\text{insist}} \rangle$, $\textcircled{C}^{\text{FID}} = \langle \text{John}, e^{\text{ponder}} \rangle$; and Steven is the understood anchor of *must*.

Consider the contrast between (26) and (27):¹⁸

- (26) Susan was worried. Her daughter might be ill.
 (27) If Susan was worried, her daughter might be ill.

The second sentence in (26) can have the meaning ‘Susan was worried that her daughter might be ill’, whereas that is not available for the main clause of the conditional in (27). Why? In (26) the author can adopt FID for the interpretation of the second sentence, tacitly triggered by consideration of Susan’s state of mind in the first. Hence, in the global context for the second sentence, $\textcircled{C}_D = \langle \textcircled{C}^{\text{FID}}, \textcircled{C}^{\text{CS}} \rangle$, where $\textcircled{C}^{\text{FID}} = \langle \text{Susan}, e^{\text{worry}} \rangle$, $\text{time}(e^{\text{worry}}) < t^*$; and the most salient center $\textcircled{C}^{\text{FID}}$ is understood to anchor *might*. But FID is only introduced globally, so it cannot be merely locally introduced by the antecedent in (27). Suppose that the global anchor in (27) is $\textcircled{C}^{\text{CS}}$, and the conditional has an epistemic modal base. Then the *if* clause introduces to the top of \textcircled{C}_D the discourse center $\textcircled{C}^{\text{if}} = \langle d^{*\oplus@}, e^{\text{CS+if}} \rangle$, the interlocutors in a circumstance in which the *if* clause is true as well as all the propositions in their CG, and the context so updated serves as the local context for interpretation of (27)’s main clause. Hence, *must* can only be anchored to that hypothetically enriched doxastic perspective, not to that of Susan.

Embedding under attitudes also makes available new potential anchors. In (9) (§2), *must* is doubly embedded, and the most local (hence most salient) $\textcircled{C}^{\text{think}} = \text{patient-Banks}$ is the only attested anchor. The nurse’s report misrepresents what patient-Banks believes, even though it is a correct report of what (coreferential) player-Banks believed prior to the accident. It is Banks’ doxastic perspective in the eventuality of utterance—in the hospital after the accident, that anchors *must* here.

Can an EMA embedded under an attitude ever have an exocentric reading, one where the anchor is not the \textcircled{C} associated with the most local embedding attitude? Yes, but (so far as I can tell) only in a very restricted set of cases.

Consider (28), involving the Mastermind scenario used extensively by von Fintel & Gillies (2008:83-84), this variation suggested by Kai von Fintel (p.c.). The game Mastermind involves a

¹⁷ See Sharvit (2008), Eckardt (2015) for overviews and extensive illustrations.

¹⁸ Thanks to an anonymous reviewer, who brought this contrast to my attention.

pair of players, one of whom, the codemaker, has full access to an array of colored pegs. The other player, the codebreaker, tries to guess the colors and pattern within a limited period. Play usually involves a series of partial guesses by the codebreaker about the number of pegs of particular colors and their locations. At each turn, the knowledgeable codemaker can give positive feedback about what the codebreaker gets right. For this variation, imagine that the master codemaker Mordecai is giving a tutorial about how to be an effective codemaker. The class has been working through a mock game, with Pascal acting the role of codebreaker. Each class member, playing at codemaker, is supposed to track the information Pascal has up to that point. Mordecai says:

- (28) You're tracking Pascal's evidence. At this point in play, you should know that there might be three reds, though in fact there are only two.

Here, one understands embedded *might* to be anchored to Pascal's perspective $\textcircled{P}^{\text{Pascal}}$. But the agent for embedding *know* is the arbitrary student Mordecai is addressing, where $\textcircled{A} \neq \textcircled{P}^{\text{Pascal}}$. Is this an exocentric reading? In this case I think there is another explanation: The first sentence explicitly reminds the addressees to adopt Pascal's perspective. But so instructed, the context for the entire second sentence is such that $\textcircled{A} = \textcircled{P}^{\text{Pascal}}$, an FID-like global shift, explaining the anchoring of *must* in the local context triggered by *know* to its subject's "denotation" Pascal.

But now consider (29) and (30):

- (29) John hasn't realized that it must be raining.
 (30) Has John realized that it must be raining?

Both have the factive attitude predicate *realize* in their main clause. John's failure to realize in (29) implicates that so far as he knows it isn't raining, and the interrogative (30) implicates that so far as the interlocutors know John may not have so realized. But these implications are inconsistent with taking the anchor for *must* to be $\textcircled{e}^{\text{realize}} = \langle \text{John}, e^{\text{realize}} \rangle$, i.e. with taking John to be the agent whose actual epistemic state (plus relevant evidence) entails that it's raining. To get the attested interpretation, we have to anchor *must* not to $\textcircled{e}^{\text{realize}}$, but to $\textcircled{*}$ or $\textcircled{\text{CS}}$, a true exocentric reading.

We can predict this reading on the present account. As usual with factives, the complement of *realize* "projects"—it is understood to be true in the common ground. But if this is the case, then the anchor for *must* must include $\textcircled{\text{CS}}$. This would be the case even if, say, we removed negation from (29), so that John, like the interlocutors, is committed to the complement's truth: Then the anchor would be $\textcircled{\text{CS}+\text{realize}} = \textcircled{*+\text{John}}$. Thus, even though John is precluded from being part of the anchor in (29) and (30), on pain of inconsistency, that still leaves the anchoring presupposition satisfied because *realize* is factive.

In such cases, we cannot take the most salient discourse center $\textcircled{e}^{\text{realize}}$ as antecedent for *must*, lest this result in an anomalous interpretation. This illustrates the claim I made in §2 that the indexical perspective presupposition of an EMA is just an instance of anaphora more generally, where anaphora resolution very strongly tends to pick out the most salient available antecedent which yields a pragmatically acceptable interpretation.

But it is at best awkward to force an exocentric reading of an attitude-embedded EMA when there is no salient alternative discourse center already in \odot_D .¹⁹ Consider the following example, discussed by Anand & Hacquard (2013:23):

- (31) [Scenario: Nobody among us has had access to the information in this filing cabinet, but we know that it contains the complete evidence (including possibly forged evidence) about the murder of Philip Boyes and narrows down the set of suspects. We are betting on who might have killed Boyes according to the information in the filing cabinet.
Harriet, who is innocent, says:]
I think I might have killed him. [Kratzer 2009:33]

I find Harriet’s utterance in (31) infelicitous, awkward at best in the context given. But Anand & Hacquard take (31) to have as one reading that in (31’), anchored to the file cabinet information:

- (31’) In all worlds compatible with Harriet’s beliefs [S], there is some world compatible *with the filing cabinet’s information* in which Harriet killed Boyes.

(31’) is the only interpretation consistent with Harriet’s knowledge that she’s innocent. But the present account predicts that that reading is unavailable, or available only through forced accommodation.

Here $\odot^* = \odot^{think} = \langle \text{Harriet, SpeechTime} \rangle$. Just as in (12), where Monk’s available evidence doesn’t include information about whether the blood of the dead girl shows traces of poison, the informational content of the filing cabinet is not something Harriet has access to in the scenario described in (31). Hence, $E^{vid}(\langle H, ST \rangle)$ includes the proposition that the cabinet contains all the evidence known to the investigators, but none of that evidence itself; and in particular, it includes no information about who the suspects might be. Since Harriet knows all this, $E^{vid}(\langle H, ST \rangle) \subseteq \text{Dox}(\langle H, ST \rangle)$. But then $f(\langle H, ST \rangle) = E^{vid}(\langle H, ST \rangle) \cup \{p \mid \text{Dox}(\langle H, ST \rangle) \subseteq p\}$ —the modal base for *might* (21)—equals $\{p \mid \text{Dox}(\odot) \subseteq p\}$, so that $\cap f(\langle H, ST \rangle) = \text{Dox}(\odot)$. Then the domain for *might* is the set of most normal worlds in Harriet’s doxastic state. This modal base would allow Harriet to truthfully assert *I think I might be one of the suspects*, but, given that she knows she’s not the killer, not (31) *I think I might be the killer*. In the scenario, there is nothing that would lead interlocutors to naturally update \odot_D prior to Harriet’s utterance to include $\odot^{according-to-the-f-c}$, so the exocentric reading is not readily available. The only way I can understand (31) as (31’) is to accommodate ‘I think that *according to the evidence in the filing cabinet* I might be the killer,’ as a way of making sense of what Harriet has said.

In this connection, it is interesting to consider EMAs that get their anchors via modal subordination (Roberts 1989, 2021):

- (32) Local meteorologists believe that a hurricane is in the offing.
It might hit the coast, or it might veer out to sea.
I myself doubt this forecast.

¹⁹ Though see Dowell (2011) for discussion of a variety of other examples.

The speaker describes a hypothetical situation, the hurricane supposedly in the offing, and then continues to consider the implications of its possible truth, adopting the irrealis scenario as the domain restriction on subsequent modals. We interpret it thus because *it* in the second sentence requires an antecedent, the only plausible one being the indefinite hurricane that takes narrow scope under *believe*. This irrealis scenario affords an antecedent for the modal base *f* for *might* in the disjuncts, making them each modally subordinate to the content of the complement of *believe*: The first disjunct means ‘if a hurricane is in the offing, it might hit the coast’, the second ‘if a hurricane is in the offing, it might veer out to sea’. In turn, subordination to that irrealis content pragmatically automatically adds the subordinating anchor $\textcircled{C}^{believe}$ from the first sentence to the local \textcircled{C}_D for each disjunct of the second, so that the set of local meteorologists can anchor *might*, yielding the intuitive interpretation. That this is the intended anchor and not \textcircled{C}^* is made clear by the speaker’s subsequent denial of the truth of that content. And unlike in (31), there is no embedding *I think* to suggest that the hurricane scenario is consistent with the speaker’s epistemic state.

I should note, because the dynamic pragmatics in §2.2 predicts it, that we find donkey anaphora with doxastic anchors, so that in (33) the contestants might draw their conclusions on very different grounds:

(33) Every contestant₅ thought_{e'} that his₅ must_{<d₅,e'>} be the best performance.

Thus, the restriction of doxastic anchors to the set of discourse centers as described in §2 predicts the range of variability in anchoring attested in the literature.²⁰

4.2 The epistemic/root distinction and Cinque’s puzzle

EMAs as a class are distinguished from other types of modal auxiliaries, called collectively the **root modals**, in several ways. Some of the most striking and consistent differences are reflected in Cinque’s (1999) puzzle, characterized by Hacquard (2010) as follows:

- a. Epistemic modals are speaker-oriented or (when embedded under an attitude predicate) attitude holder-oriented, while root modals are subject-oriented.
- b. The evaluation time (ET) of epistemic modals is the speech time (ST), whereas that of a root is provided by the tense of the clause in which it occurs.

And the further cross-correlation:

- c. When a modal is interpreted as speaker- or attitude-holder-oriented, it is evaluated at ST or attitude time, whereas when it is subject-oriented it is evaluated at the temporal location given by the tense of the clause in which it occurs.

²⁰ An anonymous reviewer points out that similar examples involving telescoping also appear acceptable:

(i) Every contestant worried about the future. He might be kicked off the show and sent home empty-handed. It would be intolerable.

One might understand telescoping as a kind of zooming in on the arbitrary instance of the quantificational domain, in this case licensing the arbitrary $\textcircled{C}^{\text{worry}}$ to serve as anchor for *might*. But since I know of no fully satisfactory account of telescoping at this time, this is just an intuition.

Related to these generalizations is the observation that epistemic modals appear to take wide scope over other operators in the clause in which they occur, including aspect, tense, and negation, while root modals do not. This tendency is illustrated with respect to tense in Hacquard's (2010) (34) with French *pouvoir*, and its English counterpart (35) with *might*:

- (34) Jean a pu prendre le train
 Jean has could take the train
 Epistemic interpretation: 'it is possible given what we now know that Jean took the train at some past time'
 Root interpretation: 'at some past time, given Jean's circumstances then, it was possible for him to take the train [and he did]'
- (35) Jean might have taken the train
 Epistemic interpretation: 'it is possible given what we now know that Jean took the train at some past time'
 Root interpretation: 'at some past time, given Jean's circumstances then, it was possible for him to take the train [and he did]'

Each of these modals has both an epistemic and a root interpretation. In the root, the temporal location of the possibility is the past time given by the past tense of the clause. But the epistemic cannot have that reading, and is only understood to pertain to a possibility present at the time of utterance.

Considering the variability in anchoring considered in §4.1, we can see that neither of the Cinque generalizations pertaining to EMAs in (a) and (b) is quite correct. First, it isn't that epistemic modals are always either speaker-oriented or local attitude-holder oriented. This ignores interrogative flip, FID, and third person exocentric anchors, the latter pragmatically given (29)/(30) or via modal subordination (32). And it isn't that the evaluation time of an epistemic modal is either speech time or the event time of an embedding attitude verb; see (23) above, and Rullmann & Matthewson (2018). Rather, the understood evaluation time of the EMA proposition is that of the attitude time of the anchoring agent. So corrected, these generalizations and the cross-correlation in (c) are immediately predicted by the present indexical account of EMAs.

EMAs are indexical, presupposing a discourse center anchor. What matters is the contextual availability of discourse centers and their relative salience. One might say that English EMAs have no tense.²¹ Rather, the indexically given situation-time at which the anchoring doxastic

²¹ I suspect that epistemic modals generally do not take semantic tense, but always get their temporal interpretation indexically. But that does not preclude their "wearing" tense. Fălăuș & Laca (2021) discuss cases where Romance epistemic modals bear past morphology, but there 'past' seems to only apply to the interpretation of the modal's prejacent. They say, "Whatever the right analysis for these cases turns out to be, it seems clear that an appropriate context for the . . . Romance examples involving modals bearing imparfait morphology requires licensing by an implicit past attitude. If this is the case, the label "past temporal perspective" is misleading as applied to them: the temporal perspective is actually simultaneous ("present"), but it is simultaneous to a past attitude, as it is the case in overt sequence of tense contexts." That is, the past-ness of the epistemic modal comes from the anchoring perspective, not from the tense it bears. Thanks to Paula Menéndez-Benito (p.c.) for pointing out their relevant work. Of course, many more languages need to be considered before one can generalize the claim about epistemic

perspective obtains serves as the evaluation time of the modal proposition, given by the same presuppositional anchoring mechanism that yields the “orientation” to the speaker (default) or other salient center (e.g., embedding agent).

In contrast, there is no evidence that root modals are indexical in this way. Unlike EMAs, root modals are compositionally related to subjects at most indirectly,²² and they get their evaluation time from the tense of the clause in which they occur.

Hacquard’s theory of the root/epistemic modal distinction:

Hacquard (2010) provides an essentially syntactic account of the differences between EMAs and root modals, relying largely on claims about differential scope at the syntactic level of representation LF. Broadly speaking, she wants to explain why EMAs tend to take wide scope with respect to the tense, aspect, and any negation in the clause in which they occur.

The essential features of Hacquard’s complex account are two: First, EMAs and root modals occur in different locations in LF, the EMA position taking wide scope over tense and aspect, the root position narrow. Second, she assumes that any modal—EMA or root—must be locally bound in LF by the first suitable c-commanding operator; and the only suitable operators binding modals are **ASP** (Aspect) and **ASSERT**, the latter an illocutionary operator only occurring at the highest level in main clauses.

Since the EMA position has wider scope than the ASP in its own clause, if a modal occurs in the higher position in a main clause, it can only be bound by **ASSERT**, while if it occurs in a clause embedded under an attitude, the modal is locally bound by the ASP of the embedding clause. The binding effectively anchors the EMA to the event associated with the binder: the speech event for **ASSERT**, the event in which the attitude obtains for an embedding ASP. Speech events are associated with an attitude, via the CG, so all possible EMA binders are associated with attitudes, and thereby indirectly with the propositional content of the attitude. This content (somehow) affords an EMA its modal base, while it gets its temporal value from the binder’s event time. Further, since an attitude event always has an agent, with the propositional content that of the agent’s attitude, this binding effectively ties the EMA to that agent’s perspective at event time.

In contrast, a modal in root position is always locally bound by the ASP of the clause in which it occurs, which needn’t be an attitude event. Its time is that of the binding event. It gets its interpretation compositionally, with a circumstantial modal base given by the circumstances associated with the binding event.

This is an elegant conception, and does a good job of accounting for Cinque’s puzzle as Cinque and Hacquard pose it (above). Assume for the purposes of discussion that the root modal account is basically correct. The essential problem for the account of EMAs is that the obligatory syntactic

modals being tenseless. But insofar as epistemic modals are indexical in the sense defined here, that is what we might expect.

²² It is clear that root modals do often bear a special relationship to the subject of the clause in which they occur. Whether this is direct (via control) or indirect (via raising) is controversial, though the latter view currently seems dominant. See the literature summarized in Portner (2009:§4.3.2:187ff).

binding makes a number of incorrect empirical predictions, and misses what I take to be an essential difference between EMA and root modals: the indexical anchoring of the EMAs.

First, note that in those cases where Hacquard makes the correct predictions about EMA interpretation, by binding matrix EMAs with her ASSERT and embedded EMAs with ASP of the embedding clause, the account in §§2 and 3 above makes the correct predictions as well, anchoring with the global discourse center $\textcircled{\text{CS}}$ instead of ASSERT or, in the embedding cases, with the locally available $\textcircled{\text{pred}}$. For example, in (34) and (35) considered out of the blue (with no prior context), we understand the anchor to be $\textcircled{\text{CS}}$, the interlocutors in the speech event. The epistemic modals *pouvoir* and *might* are not bound at LF but take their temporal value from the event-time of the anchoring center, the speech time. Thus, this account predicts the same epistemic interpretation that Hacquard derives via ASSERT and its association with the event of assertion. But even if one were to grant the use in LF of an operator like ASSERT,²³ the two accounts make very different predictions in many of the cases we have discussed above. In particular, Hacquard's account fails to predict the correct interpretations in just those cases where the above version of Cinque's Puzzle falls down, as already noted.

For example, suppose that we were to extend her account to include operators for other types of illocutionary force, e.g. QUERY and DIRECT. Under QUERY we could make the default anchor the addressee at speech time, to yield interrogative flip. But as we saw in the instruction (25), whether the main clause is imperative or declarative the anchoring of *might* in the antecedent of the conditional is to the addressee at some future time, as a function of the pragmatic function of instructions (to realize some action in the future); so the illocutionary operator itself won't help in such cases. Similarly, one might introduce yet another operator for FID, and somehow anchor that to the relevant third party agent. But this is just a further stipulation, not in itself explanatory. And what about true exocentric readings of EMAs under attitudes in (29)/(30), or cases involving modal subordination (32)? Note that the declaratives in (32) are used to assert something, so on Hacquard's account these would have ASSERT in the matrix clause, incorrectly predicting a speaker-anchor. All these examples provide evidence that you cannot derive the intended EMA anchoring from surface syntax and semantics or their illocutionary force alone.

Besides problems with non-local anchors, Hacquard's local binding predicts incorrect truth conditions in some other kinds of cases. For example, because of her treatment of ASSERT as a doxastic attitude, Hacquard can't distinguish *John is the murderer* from *I think John is the murderer*. This predicts that the following should be equivalent:

- (36) John is the murderer, but it's possible that he's not the murderer.
(37) I think John is the murderer, but it's possible that he's not the murderer.

But (36) is a contradiction, whereas (37) is not.

Here is the essential difference between the present account and Hacquard's: She would use binding of the EMA with wide scope in LF (presumably in universal grammar) to explain the fact that across languages EMAs tend to take wide scope over any other operators in the clause in

²³ about which I have serious reservations; see Roberts (2018).

which they occur (Hacquard 2010), including tense, aspect and negation. But on the present account, EMAs, like personal indexicals, take their antecedents not by binding at LF, but by anchoring to distinguished elements available in the context of utterance. We predict the wide scope effects, insofar as they are correct, with no need for syntactic stipulation, precisely because EMAs are *indexical*. So, for example, *pouvoir* (34) and *might* (35) may occur in the same position at LF for both the epistemic and root interpretations; the root will take its temporal interpretation from the local tense, while the epistemic will receive its indexically. Only when the anchoring discourse center takes narrow scope under some operator in the sentence in which the EMA occurs does the EMA itself take narrow scope. We saw this above in (33), where *must* takes narrow scope relative to the quantificational subject *every contestant*: it is the arbitrary contestant in the past eventuality denoted by the main clause which anchors the EMA, so *must* takes narrow scope relative to *every* and is (indirectly) affected by the past tense of the matrix clause because the latter's event time is the contestant's thinking-time. Otherwise, just like personal indexicals in English (Kaplan 1977), so long as an EMA is globally anchored, then no matter how deeply it is syntactically embedded, it will always seem to take wide scope over all semantic content in the utterance: It is global satisfaction of the anchoring presupposition that gives the effect of wide scope, and presumably this is so across languages.

An EMA can be shifted to the past, as we saw in (23), but this is because the adverbial *given what we knew at that time* explicitly shifts the perspective to that of the interlocutors at a prior time, not because of the past tense of the main clause. So *pace* Rullmann & Matthewson (2018), the EMA in this example does not take narrow scope relative to tense. In contrast, deontic modals are like non-modal verbs in getting their tense from the clause in which they occur and frequently taking a subject argument.

About negation, as Hacquard (2013) herself concedes, not all EMAs take wide scope relative to negation. Palmer (1979) argues that all modal auxiliaries, not just EMAs, take arbitrary fixed scope relative to negation, in some cases wide scope (epistemic *must*, *may* and *might*; deontic *must*, *should*, *ought*; dynamic *will/would*), in others narrow (epistemic *need* and *can*; deontic *may*; deontic and dynamic *can*, *could*). This can vary from language to language, so that German EMAs can vary in fixed scope from dialect to dialect (Kratzer, p.c. 1986). But this scope is lexically fixed, and arguably not a function of a special position in LF.

4.3 Yalcin's (2007) puzzle

Yalcin (2007) observes that pairs of examples like the following (his (1), (2)) appear to pose problems for the usual semantics of epistemic modality:

- (38) #It is raining and it might not be raining.
 (39) #It is raining and possibly it is not raining.

These are “epistemic contradictions”, said to be defective in a way intuitively parallel to Moore's paradox (Moore 1993) (Yalcin's examples (5), (6)):

- (40) It is raining and I do not know that it is raining.

(41) It is not raining and for all I know, it is raining.

Yalcin points out that the paradox is usually treated as involving a “pragmatic conflict”: “in making an assertion in a normal discourse context, one usually represents oneself as knowing what one says”, so each of these examples represents the speaker as “both knowing something and also as knowing that one does not know it”. Thus if *might* ϕ means ‘for all I know ϕ is true’, we get the same pragmatic conflict in (38) and (39).

Yalcin then introduces examples like (43),²⁴ where (38) is embedded under *suppose*, and explores the contrast with examples like (42) with *believe* instead of *might*:

(42) Suppose it is raining but you don’t believe it is.

(43) #Suppose it is raining but it might not be.

He points out that we cannot just “piggyback” an explanation of the embedded examples on that for the Moore’s paradox examples, because the Moore-type examples (40) and (41) (Yalcin’s (14) and (15)) are “perfectly acceptable in the embedded contexts just described”:

(44) Suppose it is raining and I do not know that it is raining.

(45) Suppose it is not raining and for all I know, it is raining.

Yalcin offers parallel examples involving *might* in the antecedents of conditionals, where, once again, there is a marked contrast with examples involving *know* instead of *might* (with (46) like Yalcin’s (16), (47) his (13)):²⁵

(46) If it is raining and I don’t know that it is raining, then...

(47) # If it is raining and it might not be raining, then...

Yalcin describes the situation thus (p.987ff):

. . . Like Moore-paradoxical sentences, epistemic contradictions are not assertable; but unlike Moore-paradoxical sentences, they are also not *supposable*, not *entertainable as true*. . . Evidently there is no coherent way to entertain the thought that it is not raining and it might be raining.

That suggests that we should drop the supposition that the two conjuncts actually are compatible. If we take it instead that $\neg\phi$ is truth-conditionally *incompatible* with $\diamond\phi$, then we will have a ready explanation for our inability to entertain their conjunction. If there simply is no possible situation with respect to which $(\neg\phi \ \& \ \diamond\phi)$ is true, then that explains why it is so hard to envisage such a situation. The conjunction is just semantically a contradiction. But although this line of explanation covers our intuitions about epistemic contradictions in embedded contexts, it comes at an unacceptably high

²⁴ (43) is Yalcin’s (8) with *but* substituted for his *and*, and the final *raining* elided. I take it that this is more natural sounding, and the change makes no difference to the logic of the examples.

²⁵ Rich Thomason (p.c. to Bill Harper, brought to my attention by Nate Charlow) noticed around 1975 that “conditionally entertaining that P is not conditionally entertaining that KP” (Rich’s characterization, p.c.), offering as an example *If my wife were cheating on me, I wouldn’t know it*, subsequently much cited in the literature.

price. If $\neg\phi$ and $\diamond\phi$ are contradictory, then the truth of one entails the negation of the other. On ordinary classical assumptions, this means that $\diamond\phi$ entails the negation of $\neg\phi$ — that is, it means $\diamond\phi$ entails ϕ . But that result is totally absurd. It would imply that the epistemic possibility operator \diamond is a factive operator, something it very clearly is not.

...

So it appears we face a dilemma.

- $\neg\phi$ and $\diamond\phi$ should be modelled as having incompatible truth conditions, in order to explain why it is not coherent to entertain or embed their conjunction; but
- $\neg\phi$ and $\diamond\phi$ should be modelled as having compatible truth conditions, in order to block the entailment from $\diamond\phi$ to ϕ .

Yalcin argues that such examples cannot be addressed within the standard relational semantics of the type developed by Kratzer, and instead offers an alternative, “domain semantics” account based on acceptance. But I will argue that on reasonable assumptions about the meanings of the expressions involved, these examples are not problematic for the indexical Kratzerian account proposed here, and in fact are exactly what it would lead us to predict. In other words, Yalcin is ignoring another possible resolution to the problem.²⁶

For contrasts like that between (42) and (43), the key to explaining the puzzle illustrated lies in explicating the meaning of *suppose*, which in its imperative form instructs the addressee to entertain a hypothetical doxastic state in which the complement is true. To do so for (43), the addressee would have to entertain a state in which the conjoined complement is true—one in which both the proposition that it’s raining and the proposition that it might not be raining are true. But what is the anchor for *might* in the complement? Under the scope of an attitude predicate, we regularly introduce a new discourse center, the agent of the attitude at the time it is held, which is the strongly preferred anchor for any EMA in the complement (unless, as we saw in §4.1, the embedding predicate is factive or we have modal subordination—neither of which is the case here). So under *suppose* the salient doxastic center is $\textcircled{C}^{\text{suppose}}$, the hypothetical doxastic agent whose belief state entails the truth of the complement at the time the supposition is realized. In imperative (43), though the addressee is the understood agent of *suppose*, the doxastic perspective of $\textcircled{C}^{\text{suppose}}$ and that of the actual addressee $\textcircled{C}^{\text{@}}$ are distinct: We’re usually asked to suppose something we don’t already actually believe. And because the complement of *suppose* is often counterfactual, the perspective entertained by $\textcircled{C}^{\text{suppose}}$ may even be incompatible with the addressee’s actual belief state. So though they have the same agent, the addressee, $\textcircled{C}^{\text{suppose}}$ and $\textcircled{C}^{\text{@}}$ are distinct doxastic centers. The embedded *might* in (43) is anchored to $\textcircled{C}^{\text{suppose}}$, whose entertained doxastic state then has to entail both the proposition that it is raining and the proposition that it might (from her own point of view) not be raining, i.e. that it’s consistent with all she knows that it isn’t raining. But this, even more clearly than in Moore’s original examples, is an inconsistent doxastic state: one both containing only worlds in which it’s raining *and* containing some worlds in which it’s not raining. Since the addressee cannot rationally entertain such a state, these directions are anomalous, and cannot be realized. This is very different from (42), which merely requires that in the revised doxastic state the addressee’s counterfactual counterpart is confused.

²⁶ The problem has, of course, been discussed by others. For example, see Anand & Hacquard (2013). Space precludes comparing all the approaches that have been considered.

That’s the intuitive account I will offer. To formalize it, I take *suppose* to be an attitude which is true of an agent just in case they entertain the proposition p denoted by its complement. We might say that to entertain a proposition is to contemplate what would be the case were it true—the way the world would be in that case.^{27 28} As with counterfactuals generally, one doesn’t entertain the complement of *suppose* alone; instead, one is interested in what p implies *against the background of one’s other beliefs about the way the world is*. But rationally entertaining a set of propositions requires that they be consistent. Accordingly, to build up the counterfactual doxastic state to be entertained, one generally has to engage in some hypothetical belief revision, to yield a state that’s consistent with p but in other respects as much as possible like the way things actually are so far as one knows.

To capture this similarity requirement, for simplicity I adapt a variant of Heim’s (1992) $\text{rev}_\phi(c)$ (the revision of a context set c to admit of the truth of the possibly counterfactual ϕ), which she intended as a rough placeholder for an adequate account of counterfactual similarity. REVDox is the revision of a doxastic state to admit of the truth of the possibly counterfactual p :

$$(48) \quad \mathbf{REVDox} =_{\text{def}} \lambda p_{\langle s, \triangleright \rangle} \lambda \text{DOX}(\odot)_{\langle s, \triangleright \rangle} \lambda w . \\ p(w) \wedge \forall w' [p(w') \rightarrow |\{r : \text{DOX}(\odot) \subseteq r \wedge r(w)\}| \geq |\{r : \text{DOX}(\odot) \subseteq r \wedge r(w')\}|]$$

REVDox takes a proposition p and a doxastic state $\text{DOX}(\odot)$ —also set of worlds, and returns a revised doxastic state that admits of the truth of p . This is the set of worlds w in which p is true, and as many as possible of the propositions r that are true in $\text{DOX}(\odot)$ and consistent with p (i.e. s.t. $p \cap r \neq \emptyset$) are also true: at least as many p -consistent $\text{DOX}(\odot)$ propositions are true in these w as in any other p -consistent worlds w' .

The formal CHARACTER of *suppose* is:

$$(49) \quad \mathbf{CHARACTER\ of\ suppose}^{29} \\ \lambda p_{\langle s, \triangleright \rangle} \lambda x_e \lambda w . \exists s [s \leq_{\text{part}} w \wedge \text{ENTERTAIN}[x, \text{REVDox}(p)(\langle x, s \rangle)(w)]]$$

Suppose takes a proposition p , an agent a and a world w to yield a proposition that is true in w just in case there is a situation s in w in which a entertains the counterfactual doxastic perspective $\text{REVDox}(p)(\langle a, s \rangle)(w)$, which is as much as possible like a ’s doxastic state in w except that p is true. I leave the notion of entertaining a doxastic perspective undefined, relying on the intuitive characterization above.

In the examples of interest here, *suppose* is used in the imperative grammatical mood. Imperatives have realization conditions, rather than truth conditions—circumstances in which they would count as being realized because the corresponding declarative with the addressee as

²⁷ Merriam-Webster (<http://www.merriam-webster.com/dictionary/entertain>) offers this definition for *entertain* (definition 3a): “to keep, hold, or maintain in the mind”. And a direction to suppose is often followed by *Then...*

²⁸ So-called counterfactual conditionals in English themselves are only possibly counterfactual, as recognized at least since Karttunen & Peters (1979). This will be the case with *suppose* as well.

²⁹ In actual implementation, the agent is an individual concept, type $\langle s, e \rangle$. I simplify for ease of exposition.

subject would be true.³⁰ These realization conditions require that the realization time be at or after the time of issuance of the directive involving the imperative. So roughly, an imperative utterance denotes a special sort of property, type $\langle s, \langle e, t \rangle \rangle$, which in English can only be realized by the addressee in the actual world:

- (50) An **imperative** S! with LF $[s ! VP]$ addressed to $a = \text{Agent}(\textcircled{a})$ in w^* at time t^* in context D is **realized** in w^* just in case: $\exists t \geq t^* : a \in |VP|^D(w^*)(t)$.

The realization of an imperative involves the imperative property holding of the addressee at (non-past) t in the world of utterance. Then imperative *suppose*, in keeping with its general CHARACTER given above, has the following realization condition, where $s \leq_{\text{part}} w$ just in case situation s is a part of world w and $s \geq_t s'$ just in case situation s temporally overlaps or follows situation s' :

- (51) CHARACTER of imperative *suppose*:
 Given context of utterance D with situation of utterance e_j :
Presupposed content: [triggered by $!$ _{i}]
 $\textcircled{a} = \langle d_i, e_j \rangle$, $e_j \leq_{\text{part}} w^*$ ‘the addressee d_i in a situation e_j the actual world w^* ’
Semantic content: (type $\langle \langle s, t \rangle, \langle s, \langle e, t \rangle \rangle \rangle$)
 $\lambda p_{\langle s, t \rangle} \lambda w \lambda x : w = w^*$, $x = d_i$. $\exists s [s \leq_{\text{part}} w \wedge \text{time}(s) \geq_t \text{time}(e_j) \wedge$
 $\text{ENTERTAIN}(x, \text{REVDox}(p)(\langle x, s \rangle)(w))]$

The presupposition introduced by the imperative mood in (51), and what it is to realize an imperative (50), guarantee that *suppose* can only be realized in the actual world in a situation in the future by the actual addressee. Imperative *suppose* takes the proposition p denoted by its complement to yield the property $\langle s, \langle e, t \rangle \rangle$ of being an individual x (the addressee) s.t. there’s an actual eventuality e in the future in which x entertains the (hypothetical) REVDox perspective consisting of x ’s doxastic state revised with p . If such an imperative is proffered as a direction, the addressee who accepts it adopts the intention of realizing the targeted property (Portner 2007, Roberts 2022).

The CHARACTER of *suppose* does not presuppose that the complement p is *not* part of the addressee’s actual belief state. But generally we assume that a directive is not felicitous if the addressee is already realizing the intended end-state. So issuance of a directive to *suppose* p implicates that so far as the speaker knows, the addressee doesn’t already believe that p or at least isn’t already entertaining it in the current circumstances.

As usual with attitude predicates, one pragmatic consequence of proffering *suppose*, be it in declarative or imperative mood, is the introduction to \textcircled{D} of $\textcircled{suppose}$. Though the agent of \textcircled{a} is the same that of $\textcircled{suppose}$, and the realization time may be the speech time, the two centers are distinct, with distinct doxastic perspectives: $\text{DOX}(\textcircled{suppose}) = \text{REVDox}(p)(\textcircled{a}) \neq \text{DOX}(\textcircled{a})$. In the realization, the addressee entertains a distinct, possibly even incompatible belief state from her actual one.

³⁰ See Portner (2007), Roberts (2018, 2022) for details, simplified here.

In (43), the addressee is directed to entertain $\text{DOX}(\textcircled{c}^{\text{suppose}}) = \text{REVDOX}(p)(\textcircled{c}^{\text{@}})$, a belief state where the complement is true, so one in which it's raining and it might not be raining. In deriving the interpretation of the second conjunct, the most salient $\textcircled{c}^{\text{suppose}}$ anchors *might*. Thus, the state to be entertained $\text{DOX}(\textcircled{c}^{\text{suppose}})$ is one in which it is raining in all the worlds in that state (first conjunct) and in which there are some worlds where it is not raining (second conjunct, with *might* so anchored). So the addressee is directed to entertain an inconsistent belief state, explaining the infelicity of the directive: It is unrealizable, a pragmatic anomaly.

But what about (42)? Why is it felicitous? The agent of *believe* in the complement of the second conjunct is conventionally given to be its subject, the denotation of *you*. But who is that individual in this hypothetical context? It is the agent of yet another center, $\textcircled{c}^{\text{REVDOX-believe}}$, the counterfactual believer *in the revised hypothetical state being entertained*, and hence the counterfactual counterpart in that state of the addressee, the agent of $\textcircled{c}^{\text{@}}$. And in the arbitrary counterfactual (REVDOX) world in which what $\textcircled{c}^{\text{suppose}}$ supposes is true, that center has yet a different perspective $\text{DOX}(\textcircled{c}^{\text{REVDOX-believe}})$. But there is no anomaly in $\textcircled{c}^{\text{suppose}}$ entertaining a doxastic state in which *p* is true though her counterpart in that state doesn't know that *p* is true: $\text{DOX}(\textcircled{c}^{\text{suppose}}) = \text{REVDOX}(p)(\langle x, t \rangle) \neq \text{DOX}(\textcircled{c}^{\text{REVDOX-believe}})$. So $\textcircled{c}^{\text{REVDOX-believe}}$ needn't realize that it's raining in the entertained, hypothetical doxastic state. Thus, (42) is felicitous.

Supporting this account of the felicity of (42), compare (42) and (43) with (52):

(52) Suppose it's raining and for all you know it might not be.

With the explicit epistemic shifter *for all you know*, (52) contrasts sharply with (43); it is more like (42). The *you* that controls embedded $\textcircled{c}^{\text{know}}$ is the hypothetical counterpart of the addressee, and so is not the same center as the $\textcircled{c}^{\text{suppose}}$ that anchors *might* in (43).

Other forms of the puzzle:

Pretty much the same thing is going on in the conditional forms of Yalcin's puzzle, but with $\textcircled{c}^{\text{if}}$ instead of counterfactual $\textcircled{c}^{\text{suppose}}$.

But something interesting happens in certain cases when we reverse the order of the two conjuncts of the *suppose* complement in (43), or the *if*-clause in (47).³¹ In (53) and (54) the results are just as unacceptable:

- (53) # Suppose that it might not be raining and it's raining. [cf. (43), same judgment]
 (54) # If it might not be raining and it's raining, then. . . [cf. (47), same judgment]

But matters are a bit trickier in other pairs. Consider (55) and (56) from Dorr & Hawthorne (2013) (their (5a) and (5b)), where one order of the antecedent conjuncts is felicitous, the other not:

(55) #If Jack and Jill fail and they might pass, then their preparation is to blame.

³¹ Nate Charlow and Phil Kremer (p.c.) pointed out the importance of the reversed order examples, for which I am grateful.

(56) If Jack and Jill might pass and they fail, then their preparation is to blame.

and the comparable pair with *suppose*:

(57) #Suppose that Jack and Jill fail and they might pass.

(58) Suppose that Jack and Jill might pass and they fail.

Though (55) and (57) are like (47) and (43) in being unacceptable, changing the order of the conjuncts in (56)/(58) results in an acceptable conditional or supposition. Why should that be? Again, for brevity I'll focus on the *suppose* cases.

The explanation for the felicity of (58) lies in the role of aspect and aktionsarten in the dynamics of temporal update, as worked out in Hinrichs (1986), Partee (1984), and Dowty (1986) using the notion of **Reference Time**—(roughly) the contextually given interval in which we take the event denoted by a clause to occur. Using a dynamic semantics, Partee shows how by systematically updating Reference Time throughout discourse we can account for a variety of effects of context on the determination of the intended event times for clauses in the discourse. In a sequence of clauses, Reference Time is updated after each clause is interpreted. Crucially for our purposes, the way it is updated is a function of whether the clause just completed displays telic or atelic aktionsarten. In an atelic clause, the corresponding eventuality is a state or on-going process which displays a sort of homogeneity (Dowty 1986): nothing changes. In contrast, in a telic, there is a change of state. Accordingly, telics typically move along the action in discourse, so that the Reference Time for interpretation of the following clause is moved forward to an interval immediately after the change of state. But atelics do not move the Reference Time forward; a sequence of atelics will typically be understood to overlap temporally. We see the difference in the following:

(59) That afternoon it was snowing. The wind was blowing and the temperature was below freezing. John was snoring in his armchair in front of the fire.

(60) Stefan was startled awake by a loud noise in the next room. He jumped up, afraid. He called the front desk and demanded an explanation. The clerk apologized.

(59) consists of a sequence of atelic clauses (progressives corresponding to on-going processes), and it is understood as describing a single period of time, when it was simultaneously snowing, windy and below freezing, while inside John was snoring. Nothing changes, and the Reference Time (the interval corresponding to *that afternoon*) stays the same from clause to clause. But in (60) we have a sequence of telic clauses, and accordingly we understand them as describing events which occur one after the other. Partee's account uses a dynamically updated Reference Time as a bookkeeping device to track how the action moves forward as a function of aktionsart.

Now consider (58). Here we're asked to entertain a state in which initially it's possible that Jack and Jill will pass. But *then* they fail. Though the possibility described by the first conjunct is atelic, the property denoted by *fail* is telic—it describes a state in which something occurs, a change of state, so that the *then* in my paraphrase is the result of the Reference Time being moved slightly forward by the regular mechanics of Reference Time update. Moreover, the changed state after the event denoted by the second conjunct occurs is one in which (assuming

both conjuncts pertain to one and the same test) the state of possibility denoted by the first conjunct is terminated. In other words, the mere possibility in the first conjunct is compatible with the occurrence of the event denoted by the second, resulting in a new state in which the possibility no longer obtains. But this interpretation is precluded in (57): we have to first entertain a situation in which Jack and Jill fail, the Reference Time for the second conjunct moving forward to *after* that event. Then assuming that we take both conjuncts to pertain to one and the same test given on a single occasion, the situation denoted by the second conjunct is no longer possible. So the scenario described by the sequentially occurring conjunction in (57) is impossible, and the resulting utterance is anomalous. The same account explains the felicity patterns in the parallel conditional examples (55) and (56).

The order of conjuncts doesn't matter in the original 'rain' pairs—(43) and (53), (47) and (54)—because in these examples both conjuncts are atelic in aktionsarten—rain is just a process, and a possibility is just a state, and in neither does anything occur to result in a change of state. Atelic aktionsart does not move the Reference Time forward, so that in the absence of any adverbial indication to the contrary, one takes a sequence of two atelic clauses to have the same Reference Time, and hence to overlap in event time, leading to the attested pragmatic contradiction.

Dorr & Hawthorne (2013) consider a version of Yalcin's puzzle with disjunction, the disjuncts differing in telicity, and the same kind of account explains their attested (in)felicity. The same type of explanation also works for what Mandelkern (2019a) calls "Wittgenstein sentences" involving disjunction, though space precludes going through the details here.

On the analysis just given, Yalcin's dilemma fails to arise. The apparent puzzle is resolved if we take into account the complexities of doxastic anchoring under iterated attitudes. By giving them their due, we have accounted for attested interpretations without resorting to the kind of non-standard modal semantics that Yalcin adopts. The account is simple and independently motivated by what we have already said about the EMAs themselves, and by the semantics of *suppose*, with a bit of Reference Time resolution (via a well-accepted account) thrown in. And *pace* Yalcin (2007,2011) and Stalnaker (2014:139ff), an account of this phenomenon requires no expressivist treatment of modality or special posterior context.

4.4 Weak necessity

Karttunen (1972) observed an apparent weakness in epistemic *must*, reflected in the pattern illustrated by examples (61) and (62) from von Stechow & Gillies (2010) (their examples (6) and (7)):

- (61) [Seeing the pouring rain]
 - a. It's raining.
 - b. ??It must be raining.
- (62) [Seeing wet rain gear and knowing rain is the only possible cause]
 - a. It's raining.
 - b. It must be raining.

The speaker in uttering (61b) gives the impression of being in too weak a position epistemically to assert (61a), which seems odd in the context given, where the evidence for the truth of the prejacent is so direct and compelling.

One would take this to be unexpected if *must* had the force of simple logical necessity, since with unrestricted modality $\Box p$ entails p .

In their proposed explanation of this puzzle, von Fintel & Gillies (2010) take *must* to be strong, like logical necessity, but explain the apparent weakness by arguing that *must* is also evidential, in that it carries a presupposition that the evidence from which the prejacent is inferred does not directly settle its truth. In (62) the evidence for rain is indirect, and consequently (62b) is felicitous because the indirectness presupposition is satisfied. But in (61) the evidence is direct, the indirectness presupposition of *must* in (61b) fails, and the utterance is infelicitous.

Because von Fintel & Gillies' semantics for English *must* is also strong, when the indirectness presupposition *is* satisfied, *must* φ entails φ . Hence they predict that someone who asserts (62b) is committed to the truth of (62a), a claim which many, myself included, find unintuitive.

There is another problem with the strong *must* approach. von Fintel & Gillies acknowledge (2010:368) that the weakness observed by Karttunen is present in epistemic modals cross-linguistically, something which they cannot explain with indirectness:

We...see no choice but to stipulate the evidential component of *must* in its lexical semantics, and we have to leave as unsolved the mystery of why this seems to be happening with every epistemic necessity modal that we have come across.

They note that one might expect this cross-linguistic universal if, instead of basing the weakness of *must* on the indirectness presupposition, it was derived via Gricean Quantity implicature. But they reject the Gricean approach out of hand:

The problem with [the weakness implicature] derivation is that there is no plausible competitor to *must* that would carry [a stronger] meaning [of the appropriate sort]. In particular, the bare assertion of the prejacent does not convey that the truth of the prejacent is known directly or through trustworthy reports. After all, it is perfectly felicitous to say that it is raining (instead of that it must be raining) upon seeing wet rain gear [as in (62a)].

Like Kratzer (2012), Matthewson (2015,§3) agrees that EMAs are evidential—"an epistemic modal is simply a modal which cares about evidence"—and discusses a range of examples that seem to support von Fintel & Gillies' claim of indirectness for English EMAs. But she also considers epistemic modals in other languages which do not have the same requirement of indirectness of the evidence constraining the domain: This could be taken to argue that the type of evidence required may differ from language to language, with language-specific requirements presupposed by its particular epistemic modals. But that would leave us without a cross-linguistic generalization to explain the observed universality of the weakness.

A number of authors have offered arguments against von Fintel & Gillies' strong account of English *must*, including Ippolito (2017) and Sherman (2018). Lassiter (2016), Del Pinel & Waldon (2018), and Degen et al. (2019) offer carefully developed experimental evidence that in ordinary usage by native speakers *must* is relatively weak, contra von Fintel & Gillies' predictions.

But then how can we explain the pattern in (61) and (62)?

It is notable that in the debate over weakness, all of the examples considered in the literature (so far as I know) involve the assertion of bare *must* ϕ . Moreover, even when an assertion is of the form *must* ϕ , weakness is only apparent when the QUD is 'whether ϕ '. Consider this variant on (28):

- (63) [Mordecai is giving a Mastermind tutorial to apprentice codemakers, working through a mock game, with Pascal acting the role of codebreaker. All the participants know that there are exactly two red pegs on the codemaker's board.]
- Mordecai: You're tracking Pascal's evidence. What do you know about how many reds there are?
- Student: There must be two reds.
- Mordecai: Wrong! Pascal hasn't yet ruled out that the last peg is yellow.

Why is the student wrong? The question isn't 'How many reds are there?' but 'What should Pascal know about how many reds there are, given his evidence?'. Hence in (63) there is no implication in either the student's reply or Mordecai's correction that the players don't know that there are exactly two reds.^{32 33 34}

This suggests a possible explanation for the infelicity of (61b): In that case the question under discussion seems to be 'Is it raining?' or the super-question 'What's the weather like?'. In this kind of context, (61a) would be a direct, more-or-less complete answer to the QUD, so is RELEVANT to it, while (61b) would be at most indirectly RELEVANT: Providing an argument that one can infer rain from the evidence doesn't satisfactorily resolve the question; it only makes it *likely* or *plausible* that the prejacent conclusion is the answer.

³² Note that the Diversity Condition of Condoravdi (2002)/the Disparity Principle of Werner (2003,2006), requiring that the modal domain contains both worlds where the prejacent is true and those where it is false, also fail to hold in cases like (63): This condition only holds when the QUD is 'whether ϕ ' or some super-question thereof. But in such cases, Disparity/Diversity follows from the QUD, for these are the alternatives in the partition.

³³ Compare the present proposal with that of Mandelkern (2019c) on this point: His "guiding observation" is that "a claim of \lceil Might p \rceil is a proposal to make p compatible with the common ground, and to make this fact itself common ground." This is what is typically asserted in uttering a declarative *might p* in a context where the QUD is 'whether p '. But in Mandelkern's formulation, the generalization is too broad: If the student in (63) had replied instead *There might be three reds*, Mordecai would have responded *That's right!*, and neither he nor the student would have intended to suggest that the prejacent should be compatible with the common ground, with which it is in fact *incompatible*.

³⁴ An anonymous reviewer and Paula Menéndez-Benito (p.c.) point out that on the present account, "*must* can be expected to give rise to weakness implicatures relative to a shifted perspective" (PMB). I think that's the right prediction: If the QUD is 'what does Pascal know about p ' and the answer is '(according to Pascal) there must be two reds', there is an inference that Pascal is not in a position to assert that p .

In Roberts' (1996/2012) QUD framework, so long as ϕ is more directly relevant to the QUD than *must* ϕ , an interlocutor *should* proffer ϕ if she believes it to be true, thereby addressing the question more directly. The felicity of any utterance requires that it be RELEVANT to the QUD, which is to say for assertions that the utterance should contextually entail at least a partial answer to that QUD. This follows from the fact that the QUD corresponds to a discourse goal, which cooperative interlocutors are committed to achieving. In the philosophy of action, if a rational agent is committed to achieving a goal, this means that she acts so as to come as close as possible to achieving it. Just so in discourse, the sincere, cooperative interlocutor, committed to the discourse goal of resolving the QUD, should proffer the strongest answer consistent with the available evidence, always keeping in mind the commitment to Quality for the CG. Strength is not measured by simple logical entailment, but by how close one's contribution comes to resolving the QUD.

Thus, in cases where 'whether ϕ ' is the QUD and the speaker asserts *must* ϕ this straightforwardly gives rise to a Gricean Quantity implicature, driven by the requirement of RELEVANCE:

- (64) **Weakness Implicature for EMAs:** Assuming that a speaker is competent and cooperative, then if the QUD is 'whether ϕ ', asserting *must* ϕ implicates that they are not in a position to assert ϕ , that they do not know (or firmly believe) it to be true.

Most often, if one hasn't yet accepted that ϕ is true, yet is explicitly considering whether ϕ , that would be because the evidence one has isn't of sufficiently high quality to foster conviction about its truth. The evidence might be merely circumstantial; or it might involve hearsay, requiring confidence in the source of the report; or it might be based on reasoning to the best explanation, leaving open unforeseen factors. Any of these might explain why one merely asserts that there's an argument that ϕ is true, instead of just asserting it. The implication is that there is some such reason.

As to why some speakers in the context in (62) might say (62a), others the weaker (62b), I take it that this typically reflects the speaker's assessment of the strength of their evidence. They know that (62a) is stronger relative to the QUD, so that if they assert it, they do so without hedging about the grounds for doing so, as in (62b). (62b) amounts to a concession that the evidence may be inadequate, the circumstances abnormal, or that the speaker has ignored a possibility that might be relevant.³⁵

One does not expect the implicature to arise in cases like (63), where *must* ϕ is *more* RELEVANT than ϕ simpliciter: Mordecai is asking the students to report on what Pascal should be able to conclude on the basis of the information that he has so far. With respect to that QUD, *there must be two reds*, anchored to Pascal in the situation of the game at play, is *more* directly relevant than the non-modal *there are two reds*.

³⁵ A related reason one might hedge about the truth of the prejacent of an EMA has to do with what's at stake, a point emphasized by Stanley (2005) about knowledge generally. The formal CHARACTER of EMAs in §3 does not reflect the subtleties about credence that would be required to capture this distinction, but it should be kept in mind.

Recall that we've effectively built relativization to the QUD into the semantic CHARACTERS of *must* and *might* in §3: In (16) E^{vid} is defined as a function yielding the set of propositions characterizing the center's RELEVANT evidential circumstances—the evidence that bears on the QUD. In the context for (62), any evidence about the actual weather is clearly directly RELEVANT. But in (63) the actual number of reds is *ir*RELEVANT: What matters to address the QUD isn't how many red pegs there actually are but what Pascal should know about how many there might be.

To explain EMA weakness, Goodhue (2018) offers an implicature similar to (64), also based on Quantity. But because he doesn't utilize the QUD, he cannot distinguish between cases like (61), where the implicature arises, and those like (63), where it does not.

Also, consider Goodhue's (65), where the teacher's point is about how to reason to a conclusion:

(65) Teacher: If x is prime and x is even, x must be 2.

Goodhue has to claim that (65) does not involve an epistemic modal, in order to make it consistent with his thesis that “*must* φ is felicitous just in case the speaker doesn't know φ ”. But he does not offer independent reasons to take *must* in (65) to be “metaphysical”. In contrast, on the present account, taking *must* to be an EMA in (65) isn't a problem: Despite the fact that the teacher knows that the prejacent is true, the weakness of *must* φ does not arise in this case precisely because what's at issue in a mathematical proof is what one can conclude from the premises, not the truth of the prejacent in the conclusion *per se*.

In fact, it has long been argued that natural language epistemic modals have an inferential semantics. Palmer (1979:59) puts it this way (my emphasis):

It is the notion of **deduction or inference from known facts** that is the essential feature of *must*, not just the strength of commitment by the speaker.

Felicitous use of *must* φ , as reflected in its CHARACTER in (20), presupposes that there is a set of premises from which φ is proffered to follow. So we can understand *must* φ as the conclusion of an argument for φ . This is why it can be odd to say *must* φ out of the blue, as observed by many authors and characterized as follows (paraphrased from Mandelkern 2019b):³⁶

(66) **SUPPORT**: Assertion of *must* φ is “degraded” (less than perfectly felicitous) unless there is a salient argument for the proposition denoted by its prejacent φ .

³⁶ See earlier arguments along similar lines due to Stone (1994), Murray (2014), Swanson (2015), Lassiter (2016), and Silk (2016). Though the data are rather delicate, I think these authors give good evidence for something like SUPPORT as a general constraint on the felicitous use of EMAs; and Mandelkern also offers convincing experimental results. SUPPORT can be taken to be a corollary of Palmer's characterization of *must* above: a reflection of the anaphoric presupposition that there *is* a salient modal base f which yields the domain of the operator in the modal's semantic content. This is a different account than that proposed by Mandelkern (2019b); unfortunately there isn't space here to discuss his proposal in detail.

As reflected in (65) and captured in (66), there's a sense in which *must* is an indirect evidential, but not in the way that von Fintel & Gillies (2010, 2021) claim. There are two kinds of indirect evidentials (Willett 1988, Aikhenvald 2006), those based on hearsay and those based on inference. English *must* and *might* are indirect evidentials not in *presupposing* that the evidence they draw on—the domain restriction given by the presupposed modal base—is itself indirect, as in von Fintel & Gillies' account, but in that assertion of *EMA* φ itself amounts to *proffering* indirect evidence for the truth of the prejacent φ . The inferential *must* presupposes a body of relevant evidence (which may be varied in its strength and provenance) and proffers that the prejacent can be inferred from that evidence: So *must* is an inferential evidential. This, I take it, explains Rett's (2016) observation that *EMA* assertions are infelicitous in exclamatives, which generally preclude indirect evidentials.

But as (65) also illustrates, unlike with hearsay and some other kinds of indirect evidence, the fact that the truth of p is inferred does not mean that *must* p is necessarily weak. In particular, in this abstract domain, inferential derivation—offering a proof—is arguably the strongest evidence one can offer for the truth of the conclusion. Hence, given an adequate proof, *x must be 2* entails that *x is 2*.

So sometimes *must* is strong, and sometimes weak, as a function of the quality of the evidence and the argument constructed from it.

von Fintel & Gillies (2021) still insist that *must* is strong. Consider what is *prima facie* perhaps their strongest evidence for the strong-*must* account: examples like (67) (their example (6)) where Alex's defense with *only* is infelicitous when the utterance criticized involves *must*:

- (67) Alex: It must be raining.
 Billy: [*opens curtains*] No it isn't. You were wrong.
 Alex: Well, strictly speaking, I was not wrong. I was careful. #I only said it must be raining.

Since their *must* is strong, von Fintel & Gillies can explain the infelicity of Alex's *I only said* defense: his initial *must* φ entailed φ : he committed himself to the truth of the false prejacent.

But we don't need strong *must* to explain the infelicity in (67). Given our realization of SUPPORT, for Alex's first utterance in (67) to be felicitous we need to imagine it in a scenario where he is offering an argument for the prejacent. For example, it could be that he and Billy are speculating about the cause of noise on the roof of their vacation cottage, and Alex offers *It must be raining* as an inference to the best explanation. For this to be a successful explanation of the noise, the argument proffered by *must* φ must be not only valid, but sound.

However, Alex's argument is arguably *not* sound: Either he took only some of the evidence into account to inductively arrive at the best explanation he could think of, or he incorrectly ruled out a less common but still perfectly reasonable explanation for the noise (a failure of imagination), or both.³⁷ He assumed that the only reasonable explanation for the pitter-patter of water on the

³⁷ The following elaborations on the scenario are not considered by von Fintel & Gillies (2021).

tin roof was that it was raining—anything else would be abnormal (ruled out by stereotypical *g*), ignoring the possibility that the gardener was using a hose to water the wisteria along the edge of the roof. Or perhaps he overlooked available evidence: Suppose that the light was rather bright around the edges of the closed curtains, leading the more observant, skeptical Billy to open them. It is generally assumed that an agent is accountable to the available evidence in drawing their conclusions. This is why Mordecai scolds the student in (63). It's why Holmes so often scolds Watson for his poor powers of observation. It's why Mr. Monk (12) denies the truth of the young detective's *might* claim. And this is captured, following Kratzer (2020), by the factual nature of the evidential conversational background in the CHARACTER of *must* in (20). The perception of the light was in Alex's evidential situation, whether he took it into account or not.

It's not a coincidence that in (67), Billie uses the assessment *you're wrong* rather than *that's false*. The latter would be odd to my ear, because Billy is not assessing the truth of the EMA statement or the prejacent so much as complaining about the unsoundness of the underlying argument: *only said must φ* turns out to be a damning admission when *must φ* as uttered proffered an unsound argument. So (67) is not an argument for strength.³⁸

It is people or their arguments that are wrong, propositions that are false. Moreover, as we saw in (63), one can be wrong even if the prejacent of one's EMA assertion is true.

There are many other ways we can get things wrong, given the pragmatics of *must* in (20). Consider:

(68) Rip van Winkle falls asleep on March 2nd 2019 in his Manhattan apartment. He wakes up on April 17th, 2020, and the electricity has been turned off for non-payment, so he can't check the internet or charge his phone. He goes out for a walk from where he lives on Union Square up along Broadway to Times Square. The streets are nearly deserted, theaters, the shops and restaurants closed, almost no cars except the occasional ambulance with its siren blaring. The few pedestrians seem furtive and afraid, scurrying along with their dogs or groceries and staying several feet away from anyone else they encounter. Storefronts are boarded up.

What the hell!?

At first Rip thinks there might have been a natural disaster. But he sees no evidence of storm damage, and the street lights are all working. Then he decides that maybe we're at war and people are hiding in terror. Maybe atomic weaponry is being used and everyone's afraid of the fallout. Or we might have already been invaded. Rip concludes:

Yes, that's it: We must be at war.

We can't really blame Rip for coming to the wrong conclusion. It would be hard to imagine before spring 2020 that major cities all over the world would be closed down tight, all but the most essential activities on hold. When Rip fell asleep, few had any idea of what a pandemic would be like in the modern world. So some as-yet-distant war was the best explanation he could come up with. Should he have seen this other possibility?

³⁸ Lassiter (2016, §4.4) makes related arguments about examples like (67), though without assuming the CHARACTER of *must* that supports this discussion here.

If Rip vaguely knew about plagues and pandemics, maybe he took them to be so far-fetched in the modern world with its excellent standards of public health that he ruled out that possibility with the ordering source *g*. But a deserted Times Square is a pretty far-fetched scenario, so maybe he should have considered it. Or perhaps he slept through all his health classes in school and didn't even know what a pandemic was, so it wasn't a possibility he *could* consider.

As assessor of an EMA assertion, I can never determine conclusively whether the speaker is considering worlds which contain exact matches of the situation in which her evidence-gathering experience obtains. I just cannot do it, partly because I myself don't have complete access to that experience. And I cannot generally examine their reasoning to determine if the underlying argument is valid. But I can hold the agent accountable for having done an adequate job of considering all the facts that were evident and relevant to her, and for avoiding obvious falsehoods and prejudice.³⁹ If Rip utters *We must be at war* to a bystander, she could reply as in (69):

- (69) Bystander: No, Rip, there's a pandemic. But you couldn't have known.
Rip: Well, at least I wasn't wrong. #I only said that we must be at war.

Knowledge (following Kratzer 2020, Djärv 2019) requires a certain kind of acquaintance which Rip did not have in these circumstances. But EMAs don't require knowledge in that strict sense (as, e.g., Kratzer says, and I concur). So the bystander is right in saying that Rip couldn't have *known*. But that doesn't mean he was right in concluding that we must be at war. The pandemic possibility was consistent with his evidence; it just didn't occur to him.

This is how we assess EMA assertions: not so much for their truth in the purely logical terms given by their semantic content, but according to the soundness of the argument, relative to the QUD and the available evidence. Insofar as the CHARACTERS in (20) and (21) help to articulate the different factors that come into play, they are useful in understanding these responses in a way that goes beyond truth conditions.

At present, I have very limited access to data about epistemic modality in other languages. But I would argue that the way that English EMAs are characterized in (20) and (21) has features that are plausible cross-linguistically: They are anchored to a salient doxastic center (indexical) and based on RELEVANT evidence for the answer to the QUD (evidential), proffering that the prejacent follows from the evidence plus other plausible assumptions about the way the world is (inferential). Insofar as that is consistent with the character of a modal-like expression in a given language, then whatever other features that modal may have, we would predict the weakness attested by von Stechow & Gillies (2010).

³⁹ An anonymous reviewer finds that another factor in the felicity of *must* claims is whether the truth of the prejacent is particularly contentious. Perhaps. It might be that in a contentious discussion it behooves the interlocutors to exercise greater precision or caution in drawing conclusions.

4.5 Second order belief and disagreement

Yalcin (2011) and Hacquard (2010) talk about a purported problem with examples like (70):

(70) Jones believes that it might rain.

Yalcin points out that standard accounts of belief and modal semantics predict that (70) attributes to Jones a second order belief—that his belief worlds are such that his knowledge (in those worlds) does not rule out it not raining—hence a belief about his own state of mind, a result which seems counterintuitive. (70) just seems to mean that Jones takes rain to be within the realm of possibility. Yalcin has his own, non-descriptivist, expressivist approach to this problem. Hacquard (2010) proposes to avoid it by denying introspection for belief.

But (70) is not a problem for the semantics for *might* in (21). The local (and most salient) anchor for *might* is $\textcircled{\text{believe}}$, Jones' actual doxastic perspective at speech time, and the predicted truth conditions are 'Jones believes that rain is consistent with the way things are'. In this paraphrase, it sounds like the domain has more content than is semantically proffered per (21): Technically, (21) just says that rain is consistent with the set of possible worlds in the domain—call this 'the evidence'. There is nothing about Jones himself in the semantic content. It is the speaker who presupposes, in uttering (70) while respecting the presuppositions of *might*, that the modal domain is the set of worlds reflecting Jones' actual evidence and his doxastic state (the way things are so far as Jones knows). But, per usual, presupposed content is not part of 'what is said': It is only the semantic content that is attributed to Jones, so there is no attribution of second order belief.

In this connection, Stalnaker (2014:143,fn.12) points out a problem with Yalcin's (2007) expressivist theory. Yalcin accounts for shifting epistemic anchoring via an indexed information state, claiming that this state is obligatorily shifted under attitudes to the information state of the agent of the attitude. But we saw in (29) and (30) that perspective shift is not obligatory under attitudes. And Stalnaker takes issue with this obligatory shift on other grounds, with the following example:

(71) Alice: Jones believes that it might rain, but I disagree—I think we can count on fine weather all day.

If the information state relative to which *might* in the complement of *believes* is interpreted is that of Jones, then by asserting the first conjunct Alice is committed to the truth of *it might rain* in Jones' information state. But (71) strikes us as consistent with Alice believing Jones is telling the truth, so we must conclude that her disagreement is not over the proposition expressed by *it might rain* as anchored to Jones, but over what possibilities are live options. This, of course, is a particularly sophisticated exemplar of the problem of faultless agreement, one of the central arguments for Relativist approaches to the semantics of EMAs (Egan et al. 2005; MacFarlane 2005, 2011; Egan 2007; Stephenson 2007, etc.).

This example points up a respect in which the debate over faultless disagreement as it has been framed to this point begs a deeper question: What does it mean to agree or disagree, either in response to what someone has said or with what someone believes?

I argue elsewhere (Roberts 2017) that (dis)agreement isn't about the truth of the proposition expressed by the speaker in a speech act or in an attitude complement, but about a comparison of features of the belief states of the relevant parties. What the complement of *believes* in (71) tells us is that it is compatible with Jones' belief state that it will rain. And this is the sense in which Alice disagrees: she fails to have a belief state that admits of the (future) possibility of rain. On this view, agreement, wherein two agents share a belief, is not about *what an agent says* so much as about the possibility that it points to. Thus in (71) Alice disagrees with Jones in that while his belief state admits of a certain possibility, hers does not. Agreement and disagreement gauge belief states as to whether they are congruent in the relevant way. Moreover, this cannot be reduced in the general case to shared assessments of the simple truth of a proposition—'what is said'. In fact, agreeing or disagreeing with some possibility under consideration has a much richer potential than just assessment of truth—we can compare the probability, likelihood, plausibility, etc. of a given proposition across the two belief states: *Jones believes that there's a very good chance it will rain, but I disagree: I don't think it's that likely.*

Space precludes developing this argument here. What is relevant is that there is reason to think that agreement and disagreement are *not* about the truth conditions of an utterance used to express an opinion. Therefore, (dis)agreement data do not offer conclusive arguments for a particular semantics for EMA statements.

5. Conclusions and prospects

The present proposal amounts to a partial answer to question (i) posed in the introduction:

- i. What constraints, if any, are there on the character of the premise set for an EMA?

In exploring the puzzles in §4, I have argued that assuming that EMAs are indexical provides a simpler, more satisfying solution, both empirically and theoretically, than other proposals intended to deal with one or more of these puzzles, as is particularly clear in addressing variable anchoring, the epistemic/root distinction, and Yalcin's puzzle. Another of its virtues is that all of the tools used in the analysis, even the discourse centers, are independently motivated, so that nothing is ad hoc.

As we saw, there are other contextual factors at play in the determination of the premise set for an EMA, notably the QUD and the available, RELEVANT evidence, as independently explored in Moss (2015). And we saw above other ways that the QUD comes to bear on EMA interpretation, notably in driving the quantity implicature that yields the weakness of *must* in §4.4. With respect to disagreement (§4.5), I have more to say about the role of the QUD in Roberts (2017b).

An anonymous reviewer argues that one might want to adopt the indexicality of EMAs without accepting evidentiality. That may be. Or one might want to characterize evidentiality differently

than I have done in §3. However, in order to make clear predictions about how indexicality bears on EMA interpretation, one must make assumptions about the appropriate type of modal accessibility relation, taking the discourse center as its argument, to use to derive the modal base presupposed by the EMA. I have found after detailed consideration of other possible domain constraints in earlier versions of this paper that the evidential approach yields better predictions overall. Others may find a yet more suitable accessibility relation. But I hope to have conveyed the importance, when considering various assumptions about the answer to (i), of keeping in mind how they interact with the indexicality I have argued for here.⁴⁰

All this leaves question (ii) relatively open:

- ii. What is the nature of the relationship between premises and conclusion that is required for truth of the EMA statement?

As we saw in the examples analyzed above, the presuppositional content proposed here is compatible with what Willer (2015) calls a “descriptive account” of epistemic *might* and *must*, like the simple semantic content of *must* and *might* in (20) and (21), while avoiding all the problems he poses for earlier descriptive accounts. A number of non-descriptive accounts have recently been promoted as ways of dealing with these problems, including relativist proposals (like that of Egan et al. 2005), expressivist accounts (including that of Yalcin 2007), and Willer’s dynamic expressivism. But these in turn encounter their own significant difficulties, which are avoided on the present approach.

The one thing I think is clearly missing from the semantic contents proposed in (20) and (21) is a way of capturing the intuition that reaching conclusions from available evidence, as reflected in EMA assertions, often involves reasoning to the best explanation, abductive rather than deductive, something emphasized by Stone (1994). But that issue is for another day.

Meanwhile, insofar as the classical account is simpler than others proposed in the literature, and easier to integrate elegantly into the overall compositional semantics for natural languages in the truth conditional tradition, this would appear to be a point in its favor.

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⁴⁰ To my dismay, as this paper was being proofread I discovered Santorio (2010), which is relevant to the proposal herein. I will have to save for another occasion discussion of Santorio’s proposal for treating epistemic and doxastic modals as Kaplanian monsters.

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